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A Decision Framework for Course Recommendation Using Basic Uncertain Linguistic Information Soft Sets

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ABSTRACT

The aim of this paper is to provide fundamental theoretical studies on basic uncertain linguistic information soft set (BULISS). Firstly, the combination of basic uncertain linguistic information and soft set is introduced. Next, set operations and similarity measure on basic uncertain linguistic information soft sets and their properties are discussed. A novel application of basic uncertain linguistic information soft set to multi-criteria group decision making is put forward, in which the similarity measure between any two BULISSs is developed. A group decision algorithm by utilizing traditional decision procedure of soft set theory (or fuzzy soft set theory) and optimization method is given. Finally, a case study relating to curriculum recommendation is shown to illustrate feasibility and validity of the developed group decision making approach.

1. Introduction

The affection of humanities and social science courses in discipline construction, program construction and student development play an important role. At the same time, the manifestation of course effectiveness is often indirect and long-term, making it difficult to accurately measure in the short term. As a result, evaluating the affection of humanities and social science courses with uncertainties is an open problem.

Uncertainty is nowadays a general phenomenon in natural events and human activities, which would bring unpredictable consequences and potential risks. To deal with kinds of uncertainties, different types of theories have been introduced to describe and measure uncertainties. For example, the probability density functions in traditional probability theory [16] are used to the possibility of values for objects. In 1965, the concept of fuzzy sets was introduced Zadeh [34], which extended the binary choice of an element in each set to the interval [0, 1]. The notion of fuzzy sets is a generalization of traditional crisp set and is a parallel theory of the probability theory [12]. Since then, the other concepts of fuzzy sets have developed. Cornelis et al. [8] discussed fuzzy logic under the interval-

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valued numerical environment. Atanassov introduced the definition of intuitionistic fuzzy sets with membership degree, non-membership degree and hesitant degree [5; 6]. To describe the hesitation among multiple numerical level, Torra introduced the notion of hesitant fuzzy sets [27]. Zadeh considered the form of linguistic variables to present qualitative evaluation [35]. Next, to illustrate the observation(s) with credibility degree, Mesiar et al. [19] developed the concepts of Z-numbers and basic uncertain information. These types of uncertain information can be derived by using collected statistical datum or personal empirical information so that diverse uncertainties in real applications can be reflected.

The concept of basic uncertain information utilizes the certainty degree to reflect the credibility of the subjective evaluation or objective observation, which has been widely studied. By combing the notion of linguistic terms and basic uncertain information, Yang et al. [33] introduced the concept of basic uncertain linguistic information (BULI). Hence, the experience of decision maker when making subjective evaluations can be thus reflected. Next, Jin et al. [14] defined some new definitions of relative basic uncertain information, relative certainty/uncertainty degree and comprehensive certainty/uncertainty with some related measurements and analysis. Yang et al. [33] put forward the dynamic three-way multi-criteria decision procedure under the environment of BULI.

To deal with the uncertainties both in datum and in structure, Maji et al. [15] combined the concepts of soft set and fuzzy set, an extended soft set named fuzzy soft set was developed. Since there are types of fuzzy information, the combinations of soft sets and fuzzy information have been widely considered. For example, the intuitionistic fuzzy soft sets [2; 22], the interval-valued fuzzy soft sets [2; 32], hesitant fuzzy soft sets [3; 10], linguistic fuzzy soft sets [17], neutrosophic fuzzy soft sets [25], type-2 fuzzy soft sets [36], another extension [1; 26], and belief interval-valued soft set [28]. These extensions can be used to associate them with diverse cases of practical uncertainties. Zou and Xiao [38] developed a data analysis approach for incomplete information. Xiao et al. [39] built a combination forecasting based on fuzzy soft sets. Herawan and Deris [13] gave a soft set method for association rules mining. Xiao et al. [31] considered a supplier selection problem by combing FCM and soft sets. The main application field is decision making. Yang et al. [33] introduced the concept of multi-fuzzy soft sets and the extension of decision procedure. Das et al. [9] enlarged the applications of soft sets to qualitative decision-making scenarios. Muthukumar and Krishnan [21] considered the application of soft sets in medical diagnosis. Arora and Garg [4] then applied it to the selection of faculty and staff. Das et al. [10] considered the application of soft sets in environmental quality assessment. Zhao and Zhang [37] showed an application of soft sets in evaluating teaching quality.

Facing the evaluation of humanities and social science courses, it's worth noting that the combination of basic uncertain linguistic information and soft set might be a feasible solution to solve such problem. Besides, such combination has not been reported. Therefore, the concept of basic uncertain linguistic information soft set will be developed. Set operations and their mathematical properties are discussed. The application in the evaluation of the affection of humanities and social science courses will also be presented.

To realize the aims mentioned above, the rest of this paper is structured in the following. Section 2 presents some basic knowledge related to basic uncertain linguistic information and soft sets. A parallel concept of basic uncertain linguistic information soft sets is introduced in Section 3. Some operations on basic uncertain linguistic information soft sets and their properties are also discussed in this section. While Section 4 provides three fundamental solutions of group decision making with basic uncertain linguistic information based on corresponding soft set theory. In Section 5, a numerical example is shown to illustrate the developed models. The comparison analysis is given to explore the effectiveness. Finally, some conclusions and remarks are summarized in Section 6.

2. Preliminaries

In this subsection, fundamental notions and notations about basic uncertain linguistic information and soft set are prepared.

2.1 Basic uncertain linguistic information

To describe the uncertainty with certainty degree, Mesiar et al. [19] introduced the following concept of basic uncertain information (BUI).

Definition 2.1. A pair $\langle x; c\rangle$ is named as a basic uncertain information, where $x \in I$ and $c \in [0,1]$ are respectively a real number in the interval I and the degree of certainty corresponding to x.

Generally, $I = [0,1]$. $\langle x; c\rangle$ will be degenerated to a real number if c=1. By Mesiar et al. [19], it can also be derived that $\langle x; c\rangle$ is equivalent to an interval [cx, cx+1-c], which can be realized through a function denoted as ϕ between the set of BUI $J = \{\langle x; c\rangle \mid x, c \in [0,1]\}$ and the interval [0, 1], i.e., $\phi(\langle x; c\rangle)$=[cx, cx+1-c].

With the definition of BUI, the notion of basic uncertain linguistic information (BILI) is developed to associate the scenario of linguistic based evaluation. Related definitions are presented in the following:

Definition 2.2 (Herrera & Martinez [40]). Let $S = \{s_0, s_1, \cdots, s_\tau\}$ be a linguistic term set with $\tau + 1$ linguistic labels. $\psi \in [0, \tau]$ is an aggregation result of a symbolic aggregation operation. The 2-tuple set $\bar{S}$ is then defined as $\bar{S} = S \times [-0.5, 0.5)$, which can express information equivalent to ψ by using the following function:

$$\Delta : [0, \tau] \to S \times [-0.5, 0.5)$$

$$\Delta(\psi) = (s_i, \alpha), with \begin{cases} s_i, & i = round(\psi) \\ \alpha = \psi - i, & \alpha \in [-0.5, 0.5) \end{cases}$$

where round(·) represents the common round operation and α is named as a symbolic translation.

Herein, the inverse function $\Delta^{-1} : S \times [-0.5, 0.5) \to [0, \tau]$ is defined according to $\Delta^{-1}(s_i, \alpha) = i + \alpha$
By integrating Definition 2.1 and Definition 2.2, the definition of BULI can be given according to

Definition 2.3 (Chen et al. [7]). Given a 2-tuple linguistic term set $\Delta(\psi), \psi \in [0, \tau]$ and its source certainty $c \in [0,1]$. The pair $b = \langle \Delta(\psi); c\rangle$ is named BULI pair.

To compare any two BULI pairs, according to Yang et al. [33], assume that $B = \{\langle \Delta(\psi); c\rangle \mid \Delta(\psi) \in \bar{S}, c \in [0,1]\}$ is the set of all BULI pairs and three projections $E : B \to [0, \tau]$, $E_\psi : B \to [0, \tau]$, $E_c : B \to [0,1]$ such that $E(b) = \psi \cdot c$, $E_\psi(b) = \psi$ and $E_c(b) = c$, then the comparison law of BULI pairs can be given as below:

Definition 2.4（Yang et al. [33]). Assume that $b_i = \langle \Delta(\psi_i); c_i\rangle (i = 1, 2)$ are any two BULI pairs, then

$$b_1 > b_2 \Leftrightarrow (E(b_1) > E(b_2)) \vee ((E(b_1) = E(b_2)) \wedge (E_\psi(b_1) > E_\psi(b_2)));$$

$$b_1 < b_2 \Leftrightarrow (E(b_1) < E(b_2)) \vee ((E(b_1) = E(b_2)) \wedge (E_\psi(b_1) < E_\psi(b_2)));$$

$$b_1 = b_2 \Leftrightarrow (E_\psi(b_1) = E_\psi(b_2)) \wedge (E_c(b_1) = E_c(b_2)).$$

It can be proved that the order relation between BULI pairs is a total order relation.

The set operations and algebra operations given can be defined according to:

Definition 2.3. Given that $B=\left\{\left\langle \Delta(\psi_{B(x)});c_{B(x)}\right\rangle \middle| \Delta(\psi_{B(x)})\in\overline{S}, c_{B(x)}\in[0,1], x\in X\right\}$ and $B_i=\left\{\left\langle \Delta(\psi_{B_i(x)});c_{B_i(x)}\right\rangle \middle| \Delta(\psi_{B_i(x)})\in\overline{S}, c_{B_i(x)}\in[0,1], x\in X\right\}$, $i=1,2$ are three basic uncertain linguistic information sets (BULISs), then

Intersection: $B_1\cap B_2=\left\{\left\langle \min_{i=1,2}\left\{\Delta(\psi_{B_i(x)})\right\}, \min_{i=1,2}\left\{c_{B_i(x)}\right\}\right\rangle \middle| x\in X\right\}$;

Union: $B_1\cup B_2=\left\{\left\langle \max_{i=1,2}\left\{\Delta(\psi_{B_i(x)})\right\}, \max_{i=1,2}\left\{c_{B_i(x)}\right\}\right\rangle \middle| x\in X\right\}$;

Complementary: $B^C=\left\{\left(\Delta\left(\tau-\Delta^{-1}(\psi_{B(x)})\right);1-c_{B(x)}\right) \middle| x\in X\right\}$.

To describe the relations among different BULISs, motivated by the inclusion relation of basic uncertain information sets, we further introduce the inclusion relation and the equality relation between any two BULISs.

Definition 2.4 (Inclusion). $B_i=\left\{\left\langle \Delta(\psi_{B_i(x)});c_{B_i(x)}\right\rangle \middle| \Delta(\psi_{B_i(x)})\in\overline{S}, c_{B_i(x)}\in[0,1], x\in X\right\}$, $i=1,2$ are supposed to be two BULISs, then B_1 is said to be included in B_2 if and only if $\Delta(\psi_{B_1(x)})\le\Delta(\psi_{B_2(x)})$ and $c_{B_1(x)}\le c_{B_2(x)}$ for any $x\in X$, which can be denoted as $B_1\subseteq B_2$.

Similarly, one can define the relation $B_2\subseteq B_1$.

Especially, it can be said that $B_1=B_2$ if and only if $B_1\subseteq B_2$ and $B_2\subseteq B_1$.

Next, the following algebra structure would be introduced for further possible consideration:

Definition 2.5. $B_i=\left\{\left\langle \Delta(\psi_{B_i(x)});c_{B_i(x)}\right\rangle \middle| \Delta(\psi_{B_i(x)})\in\overline{S}, c_{B_i(x)}\in[0,1], x\in X\right\}$ Are assumed as any two BULI and $\lambda\in[0,1]$, then

Addition: $B_1\oplus B_2=\left(\Delta\left(\min\{\psi_1+\psi_2,\tau\}\right), \min\{c_1+c_2,1\}\right)$;

Scalar-multiplication: $\lambda B_1=\left(\Delta\left(\lambda\psi_1\right), \lambda c_1\right)$;

$\left(\lambda_1\otimes B\ \right)_1\oplus\left(\left(1-\lambda\right)\otimes B_2\right)=\left(\Delta\left(\lambda\psi_1+\left(1-\lambda\right)\psi_2\right), \lambda c_1+\left(1-\lambda\right)c_2\right)$.

2.2 Soft set

According to Molodtsov [20], given that U is a real and finite universal set and E is a parameter set, and S is a subset of E. The concept of soft set can be defined according to

Definition 2.6 Molodtsov [20]. Suppose that $P(U)$ is the power set of U, a pair (F,S) is called a soft set over U if and only if F is a mapping given by $F:S\to P(U)$.

The soft set is a parameterized family of subsets in U. For any $\varepsilon\in S$, $F(\varepsilon)$ is considered as the set of ε-elements or as the set of ε-approximate elements of (F,S). Besides, the soft set $(F,[0,1])$ can be considered as the fuzzy set given by [34].

Since both parameters set and the approximate functions are crisp in soft set theory, the notion of fuzzy soft set is extended to generalize the approximate functions as fuzzy subsets of U.

Firstly, a fuzzy set X over the universe set U is defined by a mapping $\mu_X:U\to[0,1]$, where the element $\mu_X(u)$ is the membership of the truth $u\in U$ and μ_X is called the membership function of X. Generally, a fuzzy set X can be presented according to $X=\{\mu_X(u)/u:u\in U,\mu_X(u)\in[0,1]\}$.

Then, by combining with the fuzzy sets, we have

Definition 2.7 (Maji. et al. [15]; Roy and Maji [23]). Given that $\tilde{P}(U)$ is the fuzzy power set of U

, a pair $(\tilde{F},S)$ is said to be a fuzzy soft set over U, where $\tilde{F}$ is a mapping from S to $\tilde{P}(U)$, i.e., $\tilde{F}:S\rightarrow\tilde{P}(U)$.

By Definition 2.6, the fuzzy soft set can describe the uncertainties at two levels, i.e., uncertain objects corresponding to each $\varepsilon\in S$ and uncertain degrees that the objects in $\tilde{F}(S)$.

3. Main results

In this subsection, a novel fuzzy soft set called basic uncertain linguistic information soft set (BULISS) would be developed. Operations on BULISSs and their properties will also be considered.

3.1 Basic uncertain linguistic information soft sets

Taking the evaluation of a humanities and social sciences course as an example, 4 teachers who teach the same course receive evaluations from students after the course ends, 7 indicators (seen as attributes) are designed, i.e., study supervision (A1), interest stimulation (A2), skilled in interaction (A3), rich resources (A4), content arrangement (A5), post-class communication (A6) and learning effect (A7). Since there are many unknown factors that would affect the teaching and learning process, the students would provide his/her assessments by using BULI. Let $\tau=5$, Table 1 show a series of results:

Table 1.
Assessments of project consultation given by experts

	$A1$	$A2$	$A3$	$A4$	$A5$	$A6$	$A7$
$p1$	$\langle\Delta(2.9),0.56\rangle$	$\langle\Delta(3.5),0.69\rangle$	$\langle\Delta(3.0),0.74\rangle$	$\langle\Delta(3.4),0.65\rangle$	$\langle\Delta(3.1),0.68\rangle$	$\langle\Delta(3.3),0.63\rangle$	$\langle\Delta(2.8),0.57\rangle$
$p2$	$\langle\Delta(3.2),0.64\rangle$	$\langle\Delta(2.9),0.65\rangle$	$\langle\Delta(3.4),0.61\rangle$	$\langle\Delta(2.8),0.56\rangle$	$\langle\Delta(3.5),0.75\rangle$	$\langle\Delta(3.1),0.70\rangle$	$\langle\Delta(3.8),0.69\rangle$
$p3$	$\langle\Delta(3.4),0.86\rangle$	$\langle\Delta(3.1),0.76\rangle$	$\langle\Delta(2.9),0.84\rangle$	$\langle\Delta(3.1),0.58\rangle$	$\langle\Delta(3.3),0.69\rangle$	$\langle\Delta(3.4),0.71\rangle$	$\langle\Delta(3.1),0.65\rangle$
$p4$	$\langle\Delta(3.2),0.76\rangle$	$\langle\Delta(3.3),0.64\rangle$	$\langle\Delta(3.3),0.68\rangle$	$\langle\Delta(3.6),0.77\rangle$	$\langle\Delta(3.5),0.72\rangle$	$\langle\Delta(2.8),0.77\rangle$	$\langle\Delta(3.4),0.82\rangle$

As can be shown in Table 1, when evaluating all objects under each parameter, one can obtain a novel form of fuzzy set with the following form:

$$\left\{\frac{p_1}{\langle\Delta(2.9),0.56\rangle},\frac{p_2}{\langle\Delta(3.2),0.64\rangle},\frac{p_3}{\langle\Delta(3.4),0.86\rangle},\frac{p_4}{\langle\Delta(3.2),0.76\rangle}\right\},$$

where the sign of division just represents the corresponding relation between an object and its degree of satisfaction to the parameter.

Noting that if the expert has no knowledge about one of the projects under a certain parameter, then the assessments would miss the value of the project under the parameter. Thus, such form can not only describe the uncertainty that an object satisfies a parameter, but also it can express the uncertainty that part of the objects may miss the assessment(s) under certain parameter(s).

Motivated by the concept of fuzzy soft set Maji. et al. [15], the following concept of basic uncertain linguistic information soft set can be developed:

Definition 3.1. Given that U is the universe set and E a parameter set. $S\subseteq E$ is a subset of E. Let $B^U=\{\langle\Delta(\psi);c\rangle\mid\Delta(\psi)\in\overline{S},c\in[0,1]\}$ be the set of all BULI of U, Then, a basic uncertain linguistic information soft set (BULISS) over U is defined as a pair $(\breve{B},S)$, where $\breve{B}$ is a mapping given by $\breve{B}:S\rightarrow B^U$.

According to Definition 3.1, a BULISS $(\breve{B},S)$ can be detailed by

$$(\breve{B}, S) = \left\{ \left(x, x \text{ satisfies parameter } \upsilon, \left\langle \Delta(\psi_x); c_x \right\rangle \right) \middle| \forall x \in U, \upsilon \in S \right\}. \quad (1)$$

Assume that $\upsilon = \{\upsilon_1, \upsilon_2, \cdots, \upsilon_5\}$ represents the set of parameters listed in Table 1, the representation of BULISS corresponding to Table 1 can be shown in the following:

$$(\breve{B}, S) = \left\{ \begin{array}{l} \breve{B}(\upsilon_1) = \{(p_1, \langle\Delta(2.9), 0.56\rangle), (p_2, \langle\Delta(3.2), 0.64\rangle), (p_3, \langle\Delta(3.4), 0.86\rangle), (p_4, \langle\Delta(3.2), 0.76\rangle)\} \\ \breve{B}(\upsilon_2) = \{(p_1, \langle\Delta(3.5), 0.69\rangle), (p_2, \langle\Delta(2.9), 0.65\rangle), (p_3, \langle\Delta(3.1), 0.76\rangle), (p_4, \langle\Delta(3.3), 0.64\rangle)\} \\ \breve{B}(\upsilon_3) = \{(p_1, \langle\Delta(3.0), 0.74\rangle), (p_2, \langle\Delta(3.4), 0.61\rangle), (p_3, \langle\Delta(2.9), 0.84\rangle), (p_4, \langle\Delta(3.3), 0.68\rangle)\} \\ \breve{B}(\upsilon_4) = \{(p_1, \langle\Delta(3.4), 0.65\rangle), (p_2, \langle\Delta(2.8), 0.56\rangle), (p_3, \langle\Delta(3.1), 0.58\rangle), (p_4, \langle\Delta(3.6), 0.77\rangle)\} \\ \breve{B}(\upsilon_5) = \{(p_1, \langle\Delta(3.1), 0.68\rangle), (p_2, \langle\Delta(3.5), 0.75\rangle), (p_3, \langle\Delta(3.3), 0.69\rangle), (p_4, \langle\Delta(3.5), 0.72\rangle)\} \\ \breve{B}(\upsilon_6) = \{(p_1, \langle\Delta(3.3), 0.63\rangle), (p_2, \langle\Delta(3.1), 0.70\rangle), (p_3, \langle\Delta(3.4), 0.71\rangle), (p_4, \langle\Delta(2.8), 0.77\rangle)\} \\ \breve{B}(\upsilon_7) = \{(p_1, \langle\Delta(2.8), 0.57\rangle), (p_2, \langle\Delta(3.8), 0.69\rangle), (p_3, \langle\Delta(3.1), 0.65\rangle), (p_4, \langle\Delta(3.4), 0.82\rangle)\} \end{array} \right\}.$$

Besides, the following equivalent definition of BULISS can be derived:

Definition 3.2. Assume that B^E is the set of all basic uncertain linguistic information subsets of E, a pair $(\breve{\breve{B}}, U)$ is also called a basic uncertain linguistic information soft set (BULISS), which is equivalent to the BULISS over U. $\breve{\breve{B}}$ is a mapping given by $\breve{\breve{B}} : U \to B^E$.

By Definition 3.1 and Definition 3.2, $\breve{B}(\upsilon)$ and $\breve{\breve{B}}(u)$ are said to be υ-approximate elements of BULISS $(\breve{B}, S)$ and u-approximate element of BULISS $(\breve{\breve{B}}, U)$, respectively.

Since there are two special subsets named as empty set and universal set, then the following two definitions can be extended:

Definition 3.3. A BULISS $(\breve{B}, S)$ is said to be a null basic uncertain linguistic information soft set if $\breve{B}(S) = \varnothing$ for any $\upsilon \in S$.

Definition 3.4. A BULISS $(\breve{B}, S)$ is called absolute basic uncertain linguistic information soft set if $\breve{B}(S) = B^U$ for any $\upsilon \in S$.

3.2 Set operations on BULISSs

For further applications of BULISSs, in this subsection the relations among different BULISSs would be firstly considered.

Definition 3.5. Given that $S, T \subseteq E$ and $(\breve{F}, S), (\breve{G}, T)$ are two BULISSs over U, then the set relations between two BULISSs can be defined according to:

i) $(\breve{F}, S)$ is said to be a basic uncertain linguistic information soft subset of $(\breve{G}, T)$ if and only if $S \subseteq T$ and for any $\upsilon \in S$, $\breve{F}(\upsilon)$ is a basic uncertain linguistic information subset of $\breve{G}(\upsilon)$, i.e., $\breve{F}(\upsilon) \subseteq \breve{G}(\upsilon)$, which can be denoted as $(\breve{F}, S) \subset (\breve{G}, T)$. It can also be said that $(\breve{G}, T)$ is a basic uncertain linguistic information soft superset of $(\breve{F}, S)$, i.e., $(\breve{G}, T) \supset (\breve{F}, S)$.

ii) $(\breve{F}, S)$ and $(\breve{G}, T)$ are said to be basic uncertain linguistic information soft equal if and only if $(\breve{F}, S) \subset (\breve{G}, T)$ and $(\breve{G}, T) \subset (\breve{F}, S)$ hold simultaneously.

According to Definition 2.4 and Definition 3.5, the following conclusions are direct:

Property 3.1. The basic uncertain linguistic information soft inclusion relation $\subset$(or $\supset$) is a binary relation, which satisfy the properties of Reflexive, Antisymmetric and Transitive.

Property 3.2. The basic uncertain linguistic information soft equal relation is also a binary relation, which satisfy the properties of Reflexive, Symmetric and Transitive.

Next, the complement, 'AND' and 'OR' operations on basic uncertain linguistic information soft sets can be defined according to:

Definition 3.6. The complement of a basic uncertain linguistic information soft set $(\breve{F}, S)$ is denoted by $(\breve{F}, S)^C$, which can be defined according to $(\breve{F}, S)^C = (\breve{F}^C, \neg S)$. Herein, the set not S is given by $\neg S = \{\neg \upsilon = not\ \upsilon | \upsilon \in S\}$, which is the set of the opposite of parameter $\upsilon \in S$. The mapping $\breve{F}^C$ is defined as

$\breve{F}^C : \neg S \to B^U$, $\breve{F}^C(v) = \left[\breve{F}(\neg v)\right]^C$, $\forall v \in \neg S$.

Example 3.1. Following Table 1, let $A = \{\upsilon_1, \upsilon_2\}$, $B = \{\upsilon_1, \upsilon_2, \upsilon_3\}$ be two subsets of the parameter set E, and $P = \{p_1, p_2, p_3, p_4\}$ be the set of objects, and

$$\breve{F}(\upsilon_1) = \{(p_1,\langle\Delta(2.9),0.56\rangle),(p_2,\langle\Delta(3.2),0.64\rangle),(p_3,\langle\Delta(3.4),0.86\rangle),(p_4,\langle\Delta(3.2),0.76\rangle)\}$$
$$\breve{F}(\upsilon_2) = \{(p_1,\langle\Delta(3.5),0.69\rangle),(p_2,\langle\Delta(2.9),0.65\rangle),(p_3,\langle\Delta(3.1),0.76\rangle),(p_4,\langle\Delta(3.3),0.64\rangle)\}$$
$$\breve{G}(\upsilon_1) = \{(p_1,\langle\Delta(3.0),0.74\rangle),(p_2,\langle\Delta(3.4),0.68\rangle),(p_3,\langle\Delta(3.9),0.90\rangle),(p_4,\langle\Delta(3.3),0.78\rangle)\}.$$
$$\breve{G}(\upsilon_2) = \{(p_1,\langle\Delta(3.6),0.74\rangle),(p_2,\langle\Delta(3.4),0.69\rangle),(p_3,\langle\Delta(3.9),0.84\rangle),(p_4,\langle\Delta(3.3),0.68\rangle)\}$$
$$\breve{G}(\upsilon_3) = \{(p_1,\langle\Delta(3.0),0.74\rangle),(p_2,\langle\Delta(3.4),0.61\rangle),(p_3,\langle\Delta(2.9),0.84\rangle),(p_4,\langle\Delta(3.3),0.68\rangle)\}$$

It can easily be derived that $(\breve{F}, A) \subset (\breve{G}, B)$.

Besides, we have

$$\breve{F}^C(\neg\upsilon_1) = \{(p_1,\langle\Delta(2.1),0.44\rangle),(p_2,\langle\Delta(1.8),0.36\rangle),(p_3,\langle\Delta(1.6),0.14\rangle),(p_4,\langle\Delta(1.8),0.24\rangle)\},$$
$$\breve{F}^C(\neg\upsilon_2) = \{(p_1,\langle\Delta(1.5),0.31\rangle),(p_2,\langle\Delta(2.1),0.35\rangle),(p_3,\langle\Delta(1.9),0.24\rangle),(p_4,\langle\Delta(1.7),0.36\rangle)\}.$$

When trying to obtain the approximate elements under a couple of parameters in more than one parameter set, the following 'AND' and 'OR' operations would be useful:

Definition 3.7. Let $(\breve{F}, S)$ and $(\breve{G}, T)$ be two BULISSs, the "AND" operation between $(\breve{F}, S)$ and $(\breve{G}, T)$ can be defined according to

$$(\breve{F}, S) \wedge (\breve{G}, T) = (\breve{H}, S \times T), \tag{2}$$

where $\breve{H}(\rho, \upsilon) = \breve{F}(\rho) \cap \breve{G}(\upsilon)$ and $(\rho, \upsilon) \in (S, T)$.

Definition 3.8. Suppose that $(\breve{F}, S)$ and $(\breve{G}, T)$ are two BULISSs, the "OR" operation between $(\breve{F}, S)$ and $(\breve{G}, T)$ can be defined according to

$$(\breve{F}, S) \vee (\breve{G}, T) = (\breve{H}, S \times T), \tag{3}$$

where $\breve{H}(\rho, \upsilon) = \breve{F}(\rho) \cup \breve{G}(\upsilon)$ and $(\rho, \upsilon) \in (S, T)$.

Example 3.2. Following Example 3.1, let $B = \{\rho_1, \rho_2, \rho_3\}$ and

$$\breve{G}(\rho_1)=\{(p_1,\langle\Delta(3.7),0.64\rangle),(p_2,\langle\Delta(2.7),0.58\rangle),(p_3,\langle\Delta(3.9),0.88\rangle),(p_4,\langle\Delta(4.3),0.71\rangle)\},$$
$$\breve{G}(\rho_2)=\{(p_1,\langle\Delta(2.9),0.59\rangle),(p_2,\langle\Delta(3.8),0.72\rangle),(p_3,\langle\Delta(3.1),0.66\rangle),(p_4,\langle\Delta(3.2),0.65\rangle)\},$$
$$\breve{G}(\rho_3)=\{(p_1,\langle\Delta(3.3),0.74\rangle),(p_2,\langle\Delta(3.7),0.63\rangle),(p_3,\langle\Delta(3.5),0.74\rangle),(p_4,\langle\Delta(3.7),0.69\rangle)\}.$$

Then the results of 'AND' and 'OR' operations between $(\breve{F},A)$ and $(\breve{G},B)$ given by Eq. (2), (3) can be listed in Table 2. Results corresponding to 'AND' and 'OR' operations are respectively presented in the upper and lower parts of the table.

Table 2
Results of 'AND' and 'OR' operations on $(\breve{F},A)$ and $(\breve{G},B)$

	υ_1,ρ_1	υ_1,ρ_2	υ_1,ρ_3	υ_2,ρ_1	υ_2,ρ_2	υ_2,ρ_3
p1	$\langle\Delta(3.7),0.64\rangle$	$\langle\Delta(2.9),0.59\rangle$	$\langle\Delta(3.3),0.74\rangle$	$\langle\Delta(3.7),0.69\rangle$	$\langle\Delta(3.5),0.69\rangle$	$\langle\Delta(3.5),0.74\rangle$
p2	$\langle\Delta(3.2),0.64\rangle$	$\langle\Delta(3.8),0.72\rangle$	$\langle\Delta(3.7),0.64\rangle$	$\langle\Delta(2.9),0.65\rangle$	$\langle\Delta(3.8),0.72\rangle$	$\langle\Delta(3.7),0.65\rangle$
p3	$\langle\Delta(3.9),0.88\rangle$	$\langle\Delta(3.4),0.86\rangle$	$\langle\Delta(3.5),0.86\rangle$	$\langle\Delta(3.9),0.88\rangle$	$\langle\Delta(3.1),0.76\rangle$	$\langle\Delta(3.5),0.76\rangle$
p4	$\langle\Delta(4.3),0.76\rangle$	$\langle\Delta(3.2),0.76\rangle$	$\langle\Delta(3.7),0.76\rangle$	$\langle\Delta(4.3),0.71\rangle$	$\langle\Delta(3.3),0.65\rangle$	$\langle\Delta(3.7),0.69\rangle$
p1	$\langle\Delta(2.9),0.56\rangle$	$\langle\Delta(2.9),0.56\rangle$	$\langle\Delta(2.9),0.56\rangle$	$\langle\Delta(3.5),0.64\rangle$	$\langle\Delta(2.9),0.59\rangle$	$\langle\Delta(3.3),0.69\rangle$
p2	$\langle\Delta(2.7),0.58\rangle$	$\langle\Delta(3.2),0.64\rangle$	$\langle\Delta(3.2),0.63\rangle$	$\langle\Delta(2.7),0.58\rangle$	$\langle\Delta(2.9),0.65\rangle$	$\langle\Delta(2.9),0.63\rangle$
p3	$\langle\Delta(3.4),0.86\rangle$	$\langle\Delta(3.1),0.66\rangle$	$\langle\Delta(3.4),0.74\rangle$	$\langle\Delta(3.1),0.76\rangle$	$\langle\Delta(3.1),0.66\rangle$	$\langle\Delta(3.1),0.74\rangle$
p4	$\langle\Delta(3.2),0.71\rangle$	$\langle\Delta(3.2),0.65\rangle$	$\langle\Delta(3.2),0.69\rangle$	$\langle\Delta(3.3),0.64\rangle$	$\langle\Delta(3.2),0.64\rangle$	$\langle\Delta(3.3),0.64\rangle$

From Table 2, the 'AND' and 'OR' operations on BULISSs provide two assessments of the evaluator with different attitudes when synthetically consider parameters in more than one parameter set.

As can be seen in Table 2, we can also derive the following conclusions of the two operations on BULISSs.

Theorem 3.1. Given that $(\breve{F},S)$ and $(\breve{G},T)$ are two BULISSs, then we have

$$\left[(\breve{F},S)\wedge(\breve{G},T)\right]^C=(\breve{F},S)^C\vee(\breve{G},T)^C,$$
$$\left[(\breve{F},S)\vee(\breve{G},T)\right]^C=(\breve{F},S)^C\wedge(\breve{G},T)^C.$$

Proof. See Appendix 1.

Theorem 3.2. Assume that $(\breve{F},S)$, $(\breve{G},T)$ and $(\hat{O},R)$ are three FFSSs, then

i) Associative law

$$(\breve{F},S)\wedge\left((\breve{G},T)\wedge(\hat{O},R)\right)=\left((\breve{F},S)\wedge(\breve{G},T)\right)\wedge(\hat{O},R),$$
$$(\breve{F},S)\vee\left((\breve{G},T)\vee(\hat{O},R)\right)=\left((\breve{F},S)\vee(\breve{G},T)\right)\vee(\hat{O},R).$$

ii) Distribution law

$$(\breve{F},S)\wedge\left((\breve{G},T)\vee(\hat{O},R)\right)=\left((\breve{F},S)\wedge(\breve{G},T)\right)\vee\left((\breve{F},S)\wedge(\hat{O},R)\right),$$
$$(\breve{F},S)\vee\left((\breve{G},T)\wedge(\hat{O},R)\right)=\left((\breve{F},S)\vee(\breve{G},T)\right)\wedge\left((\breve{F},S)\vee(\hat{O},R)\right).$$

Proof. See Appendix 2.

The union and intersection of two BULISSs can be defined in the following:

Definition 3.9. The union of two BULISSs $\left(\breve{F}, S\right)$ and $\left(\breve{G}, T\right)$ over the universe set U is still a BULISS $\left(\breve{H}, Z\right), Z = S \bigcup T$, and

$$\breve{H}\left(\vartheta\right) = \begin{cases} \breve{F}\left(\vartheta\right), & \vartheta \in S - T, \\ \breve{G}\left(\vartheta\right), & \vartheta \in T - S, \\ \breve{F}\left(\vartheta\right) \bigcup \breve{G}\left(\vartheta\right), & \vartheta \in T \bigcap S, \end{cases}$$

which can be denoted as $\left(\breve{H}, Z\right) = \left(\breve{F}, S\right) \breve{\bigcup} \left(\breve{G}, T\right)$.

Definition 3.10. The intersection of two BULISSs $\left(\breve{F}, S\right)$ and $\left(\breve{G}, T\right)$ over the universe set U is still a BULISS $\left(\breve{H}, Z\right), Z = S \bigcap T$, and $\breve{H}\left(\vartheta\right) = \breve{F}\left(\vartheta\right) \bigcap \breve{G}\left(\vartheta\right)$, $\vartheta \in T \bigcap S$, which can be denoted as $\left(\breve{H}, Z\right) = \left(\breve{F}, S\right) \breve{\bigcap} \left(\breve{G}, T\right)$.

It can also be obtained that the following properties of the two operations on BULISSs are valid, i.e.,

Theorem 3.3. Assume that $\left(\breve{F}, S\right)$, $\left(\breve{G}, T\right)$ and $\left(\hat{O}, R\right)$ are three FFSSs, then

(1) $\left(\breve{F}, S\right) \breve{\bigcup} \left(\breve{G}, T\right) = \left(\breve{G}, T\right) \breve{\bigcup} \left(\breve{F}, S\right)$;

(2) $\left(\breve{F}, S\right) \breve{\bigcap} \left(\breve{G}, T\right) = \left(\breve{G}, T\right) \breve{\bigcap} \left(\breve{F}, S\right)$;

(3) $\left(\breve{F}, S\right) \breve{\bigcup} \left[\left(\breve{G}, T\right) \breve{\bigcup} \left(\hat{O}, R\right)\right] = \left[\left(\breve{F}, S\right) \breve{\bigcup} \left(\breve{G}, T\right)\right] \breve{\bigcap} \left(\hat{O}, R\right)$;

(4) $\left(\breve{F}, S\right) \breve{\bigcap} \left[\left(\breve{G}, T\right) \breve{\bigcap} \left(\hat{O}, R\right)\right] = \left[\left(\breve{F}, S\right) \breve{\bigcap} \left(\breve{G}, T\right)\right] \breve{\bigcap} \left(\hat{O}, R\right)$;

(5) $\left(\breve{F}, S\right) \breve{\bigcup} \left[\left(\breve{G}, T\right) \breve{\bigcap} \left(\hat{O}, R\right)\right] = \left[\left(\breve{F}, S\right) \breve{\bigcup} \left(\breve{G}, T\right)\right] \breve{\bigcap} \left[\left(\breve{F}, S\right) \breve{\bigcup} \left(\hat{O}, R\right)\right]$;

(6) $\left(\breve{F}, S\right) \breve{\bigcap} \left[\left(\breve{G}, T\right) \breve{\bigcup} \left(\hat{O}, R\right)\right] = \left[\left(\breve{F}, S\right) \breve{\bigcap} \left(\breve{G}, T\right)\right] \breve{\bigcup} \left[\left(\breve{F}, S\right) \breve{\bigcap} \left(\hat{O}, R\right)\right]$.

Proof. See Appendix 3.

Next, we would apply the novel concept of BULISSs to multi-criteria group decision making problems with basic uncertain linguistic information.

4. Multi-criteria group decision making via basic uncertain linguistic information soft set theoretical approach

Multi-criteria group decision making can utilize the strengths of collective wisdom and comprehensive reflections of the candidates, which have become an important branch of the decision-making theory [11; 22; 24]. During the evaluation of humanities and social sciences courses, an expert would provide the assessments according to his/her experiences. Like the paper review of a journal, the expert would be asked to answer the question whether you are familiar with the scope. In this section, we would utilize the concept of BULISSs as a novel tool to handle humanities and social sciences courses evaluation problems so that cognition on unascertained cases can be reflected.

Generally, a multi-criteria group decision making (MCGDM) with basic uncertain linguistic information can be described by a set of experts $Exp = \left\{e_1, e_2, \cdots, e_L\right\}$, a set of candidate $X = \left\{x_1, x_2, \cdots, x_M\right\}$ and a set of criteria $\Pi = \left\{u_1, u_2, \cdots, u_N\right\}$. To select the right choice(s), assume that the decision information of the m-th ($m = 1, 2, \cdots, M$) candidate under the n-th ($n = 1, 2, \cdots, N$) criteria given by the l-th ($l = 1, 2, \cdots, L$) expert is denoted as $\overline{F}_{mn}^{k} = b_{mn}^{k} = \left(\Delta(\psi_{mn}^{k}), c_{mn}^{k}\right)$. The

MCGDM problem is to determine the right candidate(s) by considering the decision information with respect of the set of criteria simultaneously.

Next, the basic uncertain linguistic information soft set theoretical approaches for MCGDM problems would be introduced.

4.1 Similarity measures of BULISSs

By the similarity measure of two BULI pairs Yang et al. [33], we first introduce the following concept of similarity measure between two BULISSs:

Definition 3.11. Given that $\left(\breve{F}, S\right)$ and $\left(\breve{G}, T\right)$ are two BULISSs, then the similarity measure between the two BULISSs can be defined according to

$$sim_{BULISSs}\left(\breve{F}, \breve{G}\right) = \begin{cases} \frac{1}{|X|}\frac{1}{|S\cap T|}\sum_{\upsilon_j}\sum_{x_i} sim(\breve{F}_{\upsilon_j}(x_i), \breve{G}_{\upsilon_j}(x_i)), \upsilon_j \in S\cap T \\ 0, otherwise \end{cases}$$
$$= \begin{cases} \frac{1}{|X|}\frac{1}{|S\cap T|}\sum_{\upsilon_i}\sum_{x_i}\left[1-\frac{1}{\tau}\left(\eta\left|E(b_{ij}^{\breve{F}})-E(b_{ij}^{\breve{G}})\right|+(1-\eta)\left|E_{\psi}(b_{ij}^{\breve{F}})-E_{\psi}(b_{ij}^{\breve{G}})\right|\right)\right], \upsilon_i \in S\cap T \\ 0, otherwise \end{cases}, \tag{4}$$

where $x_i \in X$ is the element, $|*|$ represents the cardinal number of a given set *, $\eta \in [0,1]$ is a parameter.

From Eq. (4), it can be seen that the similarity measure just considers assessments given by different BULISSs with the same part of the parameters.

It can be seen that the similarity measure of BULISSs has the following properties:

Symmetry: $sim_{BULISSs}\left(\breve{F}, \breve{G}\right) = sim_{BULISSs}\left(\breve{G}, \breve{F}\right)$;

Boundedness: $0 \le sim_{BULISSs}\left(\breve{G}, \breve{F}\right) \le 1$;

$sim_{BULISSs}\left(\breve{F}, \breve{F}\right) = 1$.

Especially, if the importance of parameters is different, which can be described by using a normalized weighting vector $W = \left(w_{\upsilon_1}, w_{\upsilon_2}, \cdots\right)^T$, then the following weighted similarity measure for BULISSs is valid:

$$sim_{BULISSs}\left(\breve{F}, \breve{G}\right) = \begin{cases} \frac{1}{|X|}\frac{1}{|S\cap T|}\sum_{\upsilon_j} w_{\upsilon_j}\sum_{x_i} sim(\breve{F}_{\upsilon_j}(x_i), \breve{G}_{\upsilon_j}(x_i)), \upsilon_j \in S\cap T \\ 0, otherwise \end{cases}$$
$$= \begin{cases} \frac{1}{|X|}\frac{1}{|S\cap T|}\sum_{\upsilon_i} w_{\upsilon_j}\sum_{x_i}\left[1-\frac{1}{\tau}\left(\eta\left|E(b_{ij}^{\breve{F}})-E(b_{ij}^{\breve{G}})\right|+(1-\eta)\left|E_{\psi}(b_{ij}^{\breve{F}})-E_{\psi}(b_{ij}^{\breve{G}})\right|\right)\right], \upsilon_i \in S\cap T \\ 0, otherwise \end{cases}. \tag{5}$$

4.2 Basic uncertain linguistic information soft set based MCGDM procedure

With the notations mentioned above, by combining with some existing decision procedures, some basic uncertain linguistic information soft set theory based multi-criteria group decision making procedures would be introduced in this section.

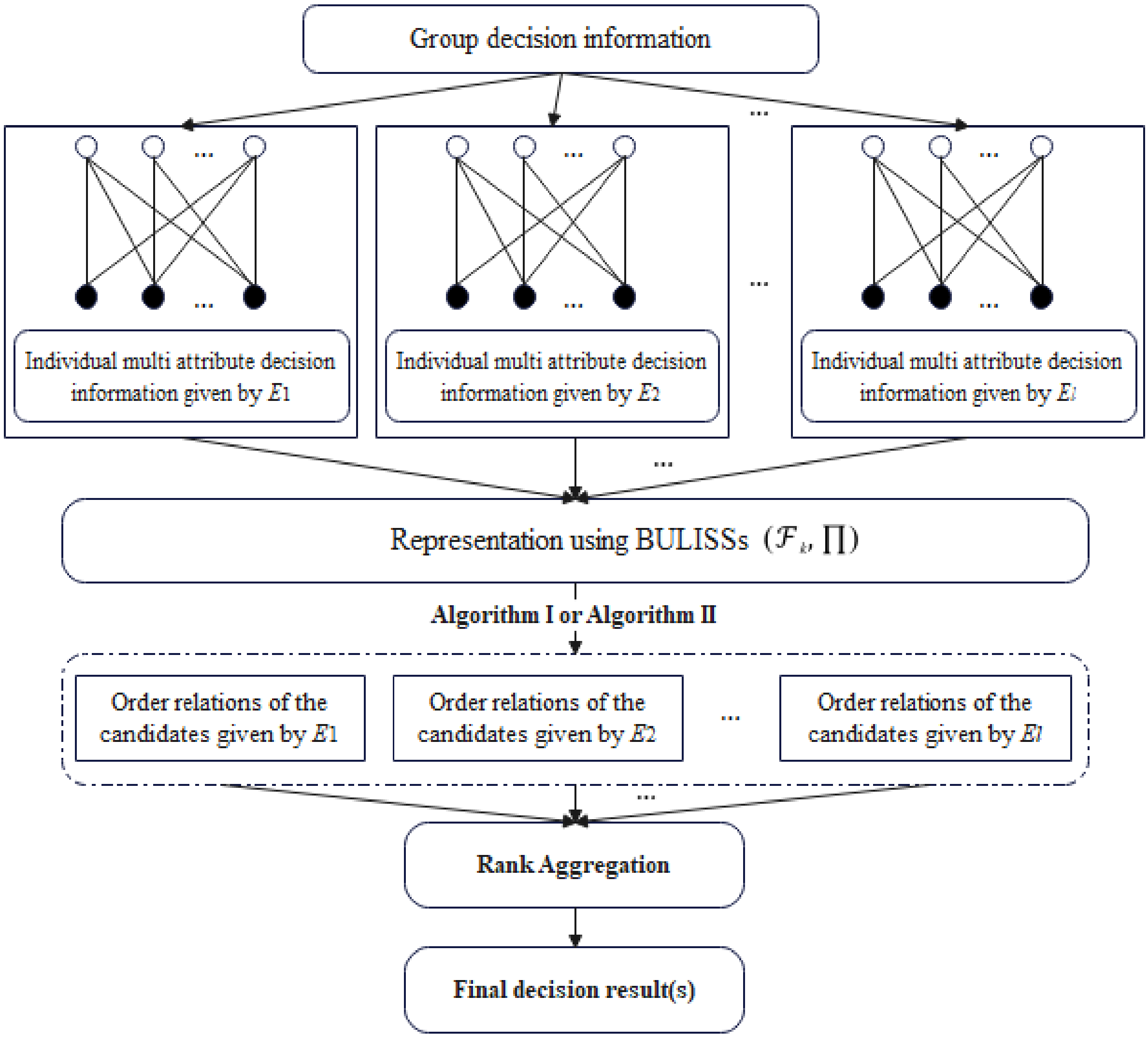

Fig.1: Graphical representation of the framework

Firstly, the fundamental decision procedures given by traditional soft set and fuzzy soft set are presented:

Algorithm I Maji. et al. [15].

①Input (F, A);②Calculate $u_m = \sum_n h_{mn}$, $h_{mn} = \begin{cases} 1, x_m \in F(u_n) \\ 0, x_m \notin F(u_n) \end{cases}$;③Decision: $u_{optimal} = \max_m u_m$.

and

Algorithm II Roy and Maji [23].

① Input $(\tilde{F}, A)$; ② Compute comparison: $r_p = \sum_{q=1}^{N} u_{pq}, t_q = \sum_{p=1}^{M} u_{pq}$, $c_{pq} = \left|\left\{e \in A : \tilde{F}_e(x_p) \geq \tilde{F}_e(x_q)\right\}\right|$; ③ Calculate comparison score $\leftarrow s_j = r_j - t_j$; ④ Decision $\leftarrow s_{optimal} = \max_j score_j$.

Total framework the developed decision procedure can be shown in the following Fig. 1.

As shown in Fig. 1, two key steps, including deriving individual order relations and rank aggregation are utilized to produce the final recommendation.

By considering the existing fuzzy soft set-based decision-making algorithms, next we would provide a basic uncertain linguistic information soft set based on MCGDM procedure.

Step 1. Collection of group decision information

As illustrated by Wang [30], the determination of fuzzy information in practical applications can be obtained according to the knowledge of human experts or various sensors.

Step 2. Basic uncertain linguistic information soft representation of the MCGDM problems

Different from existed methods of aggregating the group decision information, the proposed group decision making approach is to utilize the concept of basic uncertain linguistic information fuzzy soft sets. As a result, we first transfer the group decision information to associate with the form of BULISSs.

With the notations mentioned above, the form of BULISSs corresponding to the decision matrices given by different experts can be transformed in the following:

$$\left(F_k, \Pi\right) = \left\{\left(x, \left(\Delta(\psi_j^k(x)), c_j^k(x)\right)\right) \middle| \forall x \in X, u_j \in \Pi\right\}, k = 1, 2, \cdots, L.$$

Step 3. Determination of the weighting vector of experts in MCGDM

By using Eq. (4), the similarities between any two BULISSs in MAGDM can be calculated, which can be denoted as $sim\left(F_k, F_{k'}\right) = sim_{BULISSs}\left(F_k, F_{k'}\right)$.

For the sake of making decisions, the dissimilarities among decision information given by different experts are encouraged. As a result, the larger the similarities of a BULISS $\left(F_k, \Pi\right)$ with the other BULISSs, the smaller the weight of $\left(F_k, \Pi\right)$ will be. Assume that ω_k is the weight associated with $\left(F_k, \Pi\right)$, then we have

$$\omega_k = \frac{\sum_{k'}\left(1 - sim\left(F_k, F_{k'}\right)\right)}{\sum_k \sum_{k'}\left(1 - sim\left(F_k, F_{k'}\right)\right)}, k = 1, 2, \cdots, L. \quad (6)$$

From Eq. (6), the importance of experts can be measured. It's worth noting that the weight is a relative value, which is aiming at making a group decision.

Step 4. Order relations of the candidates given by different experts

By utilizing Algorithm II and the comparisons between basic uncertain linguistic information shown in subsection 2.1, for each BULISS $\left(F_k, \Pi\right)$, the order relation of candidates given by the k-th expert can be obtained.

For convenience, suppose that $x_{im}^k, i, m = 1, 2, \cdots, M$ represents the i-th candidate locates at the m-th position given by the k-th expert.

Step 5. Rank aggregation of orders provided by different experts

According to the weighting vector obtained in Step 3 and the ranks of candidates derived in Step 4, we would build an optimization model to produce the final ranking of all candidates with the minimum disagreement of groups.

In Wan and Dong [29], let $X = \left(x_{im}\right)_{M \times M}$ be the final aggregated ranking matrix of the groups, where x_{ij} represents the i-th candidate locates in the j-th position, then X can be determined according to the following optimization model:

$$\min \sum_{k=1}^{N} \sum_{i=1}^{M} \sum_{m=1}^{M} \left|x_{im}^k - x_{im}\right|$$

$$s.t., \begin{cases} \sum_{i=1}^{M} x_{im} = 1, m = 1, 2, \cdots, M \\ \sum_{m=1}^{M} x_{im} = 1, i = 1, 2, \cdots, M \\ x_{im} = \begin{cases} 1, if\ x_i\ locates\ in\ the\ m-th\ position \\ 0, otherwise \end{cases} \end{cases} \quad (7)$$

To contain the importance of experts in the final ranking aggregation, Eq. (7) can be modified as Eq. (8).

By Eq. (8), the final aggregated ranking matrix X considers the importance of experts and the disagreements of groups so that the total order of candidates with the minimum disagreement can be derived.

$$\min \sum_{k=1}^{N} \omega_k \sum_{i=1}^{M} \sum_{m=1}^{M} \left| x_{im}^{k} - x_{im} \right|$$
$$s.t., \begin{cases} \sum_{i=1}^{M} x_{im} = 1, m = 1, 2, \cdots, M \\ \sum_{m=1}^{M} x_{im} = 1, i = 1, 2, \cdots, M \\ x_{im} = \begin{cases} 1, if\ x_i\ locates\ in\ the\ m-th\ position \\ 0, otherwise \end{cases} \end{cases} \tag{8}$$

Note. It's worth noting that the ranking given by each expert would produce the paratactic case(s). In order to associate with such case(s), the weight(s) would also be partitioned as more than one equalized value, i.e., if there are p groups of q equalized ranks in the order of alternatives given by an expert, then the weight would be partitioned as $p \cdot q!$ different rankings so that the equalized ranks can be handled.

Step 6. Determination of the total order of candidates.

According to the solution of Eq. (8), the group decision making can be derived. One can obtain the best choice(s) according to X. In other words, by using the final aggregated ranking matrix X, one can get the top-K candidates as the recommendation.

To obtain the ranking of candidates given by each expert, one can also utilize Algorithm I according to the concept of α-level soft set (Jiang, Tang and Chen [18]) by setting the satisfactory level of decision maker.

5. Case study

5.1 Fundamental results

In this section, a humanities and social sciences courses evaluation case is shown to illustrate the application of the developed group decision making method.

The school of Marxism in university A needs to evaluate 6 courses for possible optimization includes adding/deleting credits, adding a course, deleting a course or adjusting course's attribute. The necessity of optimization on these courses is evaluated by the group of graduated students, colleagues and cross disciplinary peers, which are respectively denoted as e1, e2 and e3. For convenience, all candidates are evaluated from 5 aspects, i.e., availability (u1, the optimization is helpful for students in future study or work), accessibility (u2, the optimization can be realized in practical teaching and learning process), affordability (u3, the optimization allowed due to credit limitation and other factors), acceptability (u4, the optimization meets the needs of professional development or industry demands), and adaptability (u5, the optimization is compatible with existing curriculum system). For simplicity, the evaluation structure is named as accessibility of courses' optimization. Assume that 6 courses are respectively denoted as x1 to x6.

Table 3 shows the assessments given by three experts ($\tau = 5$). It can be seen that the assessments are totally consistent at numerical level but fluctuations are also existed. The school needs to realize a ranking of these courses so that which course(s) should be optimized can be correctly judged.

Table 3
Individual decision information given by 3 experts.

***e*1**	*u*1	*u*2	*u*3	*u*4	*u*5
*x*1	(Δ(3.5); 0.76)	(Δ(2.9); 0.65)	(Δ(3.2); 0.65)	(Δ(3.2); 0.66)	(Δ(3.1); 0.62)
*x*2	(Δ(3.3); 0.64)	(Δ(3.1); 0.76)	(Δ(3.4); 0.73)	(Δ(3.1); 0.65)	(Δ(2.8); 0.68)
*x*3	(Δ(2.9); 0.67)	(Δ(3.1); 0.72)	(Δ(3.3); 0.62)	(Δ(3.5); 0.71)	(Δ(2.8); 0.59)
*x*4	(Δ(3.2); 0.63)	(Δ(3.0); 0.73)	(Δ(3.2); 0.69)	(Δ(3.4); 0.59)	(Δ(3.1); 0.74)
*x*5	(Δ(3.1); 0.73)	(Δ(3.4); 0.73)	(Δ(3.4); 0.65)	(Δ(3.4); 0.73)	(Δ(3.5); 0.76)
*x*6	(Δ(2.9); 0.77)	(Δ(2.9); 0.58)	(Δ(3.4); 0.79)	(Δ(3.2); 0.71)	(Δ(2.8); 0.66)
***e*2**	*u*1	*u*2	*u*3	*u*4	*u*5
*x*1	(Δ(3.1); 0.67)	(Δ(3.0); 0.75)	(Δ(3.6); 0.61)	(Δ(3.3); 0.74)	(Δ(3.6); 0.70)
*x*2	(Δ(3.3); 0.60)	(Δ(3.4); 0.74)	(Δ(3.0); 0.80)	(Δ(2.8); 0.69)	(Δ(3.4); 0.67)
*x*3	(Δ(3.7); 0.82)	(Δ(2.9); 0.79)	(Δ(3.1); 0.64)	(Δ(3.6); 0.66)	(Δ(3.3); 0.74)
*x*4	(Δ(3.0); 0.62)	(Δ(3.2); 0.73)	(Δ(3.1); 0.66)	(Δ(3.3); 0.71)	(Δ(3.6); 0.64)
*x*5	(Δ(3.3); 0.71)	(Δ(3.0); 0.74)	(Δ(3.5); 0.62)	(Δ(3.1); 0.68)	(Δ(3.3); 0.63)
*x*6	(Δ(2.9); 0.65)	(Δ(3.2); 0.61)	(Δ(2.8); 0.56)	(Δ(3.5); 0.75)	(Δ(3.1); 0.70)
***e*3**	*u*1	*u*2	*u*3	*u*4	*u*5
*x*1	(Δ(2.8); 0.69)	(Δ(3.1); 0.76)	(Δ(2.9); 0.84)	(Δ(3.1); 0.58)	(Δ(3.3); 0.69)
*x*2	(Δ(3.4); 0.71)	(Δ(3.2); 0.65)	(Δ(3.3); 0.64)	(Δ(3.3); 0.68)	(Δ(3.6); 0.77)
*x*3	(Δ(2.7); 0.63)	(Δ(3.5); 0.72)	(Δ(2.8); 0.77)	(Δ(2.9); 0.56)	(Δ(3.2); 0.64)
*x*4	(Δ(3.1); 0.67)	(Δ(3.8); 0.78)	(Δ(2.8); 0.57)	(Δ(3.4); 0.65)	(Δ(3.7); 0.84)
*x*5	(Δ(3.5); 0.69)	(Δ(2.9); 0.68)	(Δ(3.2); 0.76)	(Δ(3.8); 0.69)	(Δ(3.1); 0.74)
*x*6	(Δ(2.9); 0.62)	(Δ(3.4); 0.86)	(Δ(3.5); 0.67)	(Δ(3.4); 0.67)	(Δ(3.3); 0.64)

By using the above Step 2 representation, the form of BULISSs corresponding to the decision matrix given by the three experts can be transformed into:

$$(F_1,\Pi)=\left\{\begin{array}{l}
F_1(u_1)=\left\{\frac{x_1}{(\Delta(3.5);0.76)},\frac{x_2}{(\Delta(3.3);0.64)},\frac{x_3}{(\Delta(2.9);0.67)},\frac{x_4}{(\Delta(3.2);0.63)},\frac{x_5}{(\Delta(3.1);0.73)},\frac{x_6}{(\Delta(2.9);0.77)}\right\}\\
F_1(u_2)=\left\{\frac{x_1}{(\Delta(2.9);0.65)},\frac{x_2}{(\Delta(3.1);0.76)},\frac{x_3}{(\Delta(3.1);0.72)},\frac{x_4}{(\Delta(3.0);0.73)},\frac{x_5}{(\Delta(3.4);0.73)},\frac{x_6}{(\Delta(2.9);0.58)}\right\}\\
F_1(u_3)=\left\{\frac{x_1}{(\Delta(3.2);0.65)},\frac{x_2}{(\Delta(3.4);0.73)},\frac{x_3}{(\Delta(3.3);0.62)},\frac{x_4}{(\Delta(3.2);0.69)},\frac{x_5}{(\Delta(3.4);0.65)},\frac{x_6}{(\Delta(3.4);0.79)}\right\}\\
F_1(u_4)=\left\{\frac{x_1}{(\Delta(3.2);0.66)},\frac{x_2}{(\Delta(3.1);0.65)},\frac{x_3}{(\Delta(3.5);0.71)},\frac{x_4}{(\Delta(3.4);0.59)},\frac{x_5}{(\Delta(3.4);0.73)},\frac{x_6}{(\Delta(3.2);0.71)}\right\}\\
F_1(u_5)=\left\{\frac{x_1}{(\Delta(3.1);0.62)},\frac{x_2}{(\Delta(2.8);0.68)},\frac{x_3}{(\Delta(2.8);0.59)},\frac{x_4}{(\Delta(3.1);0.74)},\frac{x_5}{(\Delta(3.5);0.76)},\frac{x_6}{(\Delta(2.8);0.66)}\right\}
\end{array}\right\}$$

$$(F_2,\Pi)=\left\{\begin{array}{l}
F_2(u_1)=\left\{\frac{x_1}{(\Delta(3.1);0.67)},\frac{x_2}{(\Delta(3.3);0.60)},\frac{x_3}{(\Delta(3.7);0.82)},\frac{x_4}{(\Delta(3.0);0.62)},\frac{x_5}{(\Delta(3.3);0.71)},\frac{x_6}{(\Delta(2.9);0.65)}\right\}\\
F_2(u_2)=\left\{\frac{x_1}{(\Delta(3.0);0.75)},\frac{x_2}{(\Delta(3.4);0.74)},\frac{x_3}{(\Delta(2.9);0.79)},\frac{x_4}{(\Delta(3.2);0.73)},\frac{x_5}{(\Delta(3.0);0.74)},\frac{x_6}{(\Delta(3.2);0.61)}\right\}\\
F_2(u_3)=\left\{\frac{x_1}{(\Delta(3.6);0.61)},\frac{x_2}{(\Delta(3.0);0.80)},\frac{x_3}{(\Delta(3.1);0.64)},\frac{x_4}{(\Delta(3.1);0.66)},\frac{x_5}{(\Delta(3.5);0.62)},\frac{x_6}{(\Delta(2.8);0.56)}\right\}\\
F_2(u_4)=\left\{\frac{x_1}{(\Delta(3.3);0.74)},\frac{x_2}{(\Delta(2.8);0.69)},\frac{x_3}{(\Delta(3.6);0.66)},\frac{x_4}{(\Delta(3.3);0.71)},\frac{x_5}{(\Delta(3.1);0.68)},\frac{x_6}{(\Delta(3.5);0.75)}\right\}\\
F_2(u_5)=\left\{\frac{x_1}{(\Delta(3.6);0.70)},\frac{x_2}{(\Delta(3.4);0.67)},\frac{x_3}{(\Delta(3.3);0.74)},\frac{x_4}{(\Delta(3.6);0.64)},\frac{x_5}{(\Delta(3.3);0.63)},\frac{x_6}{(\Delta(3.1);0.70)}\right\}
\end{array}\right\}$$

$$(F_3,\Pi)=\left\{\begin{array}{l}
F_3(u_1)=\left\{\frac{x_1}{(\Delta(2.8);0.69)},\frac{x_2}{(\Delta(3.4);0.71)},\frac{x_3}{(\Delta(2.7);0.63)},\frac{x_4}{(\Delta(3.1);0.67)},\frac{x_5}{(\Delta(3.5);0.69)},\frac{x_6}{(\Delta(2.9);0.62)}\right\}\\
F_3(u_2)=\left\{\frac{x_1}{(\Delta(3.1);0.76)},\frac{x_2}{(\Delta(3.2);0.65)},\frac{x_3}{(\Delta(3.5);0.72)},\frac{x_4}{(\Delta(3.8);0.78)},\frac{x_5}{(\Delta(2.9);0.68)},\frac{x_6}{(\Delta(3.4);0.86)}\right\}\\
F_3(u_3)=\left\{\frac{x_1}{(\Delta(2.9);0.84)},\frac{x_2}{(\Delta(3.3);0.64)},\frac{x_3}{(\Delta(2.8);0.77)},\frac{x_4}{(\Delta(2.8);0.57)},\frac{x_5}{(\Delta(3.2);0.76)},\frac{x_6}{(\Delta(3.5);0.67)}\right\}\\
F_3(u_4)=\left\{\frac{x_1}{(\Delta(3.1);0.58)},\frac{x_2}{(\Delta(3.3);0.68)},\frac{x_3}{(\Delta(2.9);0.56)},\frac{x_4}{(\Delta(3.4);0.65)},\frac{x_5}{(\Delta(3.8);0.69)},\frac{x_6}{(\Delta(3.4);0.67)}\right\}\\
F_3(u_5)=\left\{\frac{x_1}{(\Delta(3.3);0.69)},\frac{x_2}{(\Delta(3.6);0.77)},\frac{x_3}{(\Delta(3.2);0.64)},\frac{x_4}{(\Delta(3.7);0.84)},\frac{x_5}{(\Delta(3.1);0.74)},\frac{x_6}{(\Delta(3.3);0.64)}\right\}
\end{array}\right\}$$

Next, the weights of experts need to be determined.

By using Eq. (4), the similarities between any two BULISSs in MAGDM can be calculated, which can be denoted as $sim(F_k, F_{k'}) = sim_{FFSs}(F_k, F_{k'})$ by setting $\eta = 0.5$. The results can be obtained and listed in Table 4. It's a symmetric matrix in essence describing the similarities between any two BULISSs. By Table 4, F_1 and F_2 perform the highest similarity.

Table 4
Similarities between any two BULISSs in MAGDM

Similarities	F_1	F_2	F_3
F_1	1	0.9644	0.9582
F_2	0.9644	1	0.9590
F_3	0.9582	0.9590	1

Therefore, according to the Eq. (6), the weights corresponding to three experts w_1, w_2, w_3 can be obtained, i.e., $w_1 = 0.3004$, $w_2 = 0.3530$, $w_3 = 0.3466$.

According to Algorithm II and the comparison laws, the rankings of alternatives given by three experts can be produced, which are listed in the following Table 5.

Table 5
Single rankings given by 3 'expert' by Algorithm II

	*x*1	*x*2	*x*3	*x*4	*x*5	*x*6
*e*1	4(*s*1=-3)	2(*s*2=1)	5(*s*3=-7)	6(*s*4=-7)	1(*s*5=17)	3(*s*6=-1)
*e*2	1(*s*1=11)	3(*s*2=3)	2(*s*3=7)	4(*s*4=-3)	5(*s*5=-7)	6(*s*6=-11)
*e*3	3(*s*1=1)	4(*s*2=1)	6(*s*3=-15)	2(*s*4=5)	1(*s*5=7)	5(*s*6=1)

By Table 5, it's worth noting that the candidates with the same value of comparison score would be assigned with the same order. For instance, s3= s4=-7 can be derived from the decision information provided by e1, then both can be ranked in the 5-th and the 6-th position. Thus, Table 5 is firstly extended to the following Table 6.

Table 6
Single rankings given by 3 'expert' by Algorithm II

	*x*1	*x*2	*x*3	*x*4	*x*5	*x*6
*e*1	4(*s*1=-3)	2(*s*2=1)	5(*s*3=-7)	6(*s*4=-7)	1(*s*5=17)	3(*s*6=-1)
*e*1'	4(*s*1=-3)	2(*s*2=1)	6(*s*3=-7)	5(*s*4=-7)	1(*s*5=17)	3(*s*6=-1)
*e*2	1(*s*1=11)	3(*s*2=3)	2(*s*3=7)	4(*s*4=-3)	5(*s*5=-7)	6(*s*6=-11)
e_3^1	3(*s*1=1)	4(*s*2=1)	6(*s*3=-15)	2(*s*4=5)	1(*s*5=7)	5(*s*6=1)
e_3^2	3(*s*1=1)	5(*s*2=1)	6(*s*3=-15)	2(*s*4=5)	1(*s*5=7)	4(*s*6=1)
e_3^3	4(*s*1=1)	3(*s*2=1)	6(*s*3=-15)	2(*s*4=5)	1(*s*5=7)	5(*s*6=1)
e_3^4	4(*s*1=1)	5(*s*2=1)	6(*s*3=-15)	2(*s*4=5)	1(*s*5=7)	3(*s*6=1)
e_3^5	5(*s*1=1)	4(*s*2=1)	6(*s*3=-15)	2(*s*4=5)	1(*s*5=7)	3(*s*6=1)
e_3^6	5(*s*1=1)	3(*s*2=1)	6(*s*3=-15)	2(*s*4=5)	1(*s*5=7)	4(*s*6=1)

Next, according to Eq. (8), the weighting vector and Tables 5, the following optimization model can be obtained:

$$\min \frac{1}{2} w_1 \sum_{i=1}^{6} \sum_{m=1}^{6} \left| x_{im}^{k} - x_{im} \right| + w_2 \sum_{i=1}^{6} \sum_{m=1}^{6} \left| x_{im}^{k} - x_{im} \right| + \frac{w_3}{4} \sum_{i=1}^{6} \sum_{m=1}^{6} \left| x_{im}^{3} - x_{im} \right|$$

$$s.t., \begin{cases} \sum_{i=1}^{6} x_{im} = 1, m = 1, 2, \cdots, 6 \\ \sum_{m=1}^{6} x_{im} = 1, i = 1, 2, \cdots, 6 \\ x_{im} = \begin{cases} 1, if \ x_i \ locates \ in \ the \ m-th \ position \\ 0, otherwise \end{cases} \end{cases}$$

By solving the optimization model Eq. (8), the solution can be derived.

Therefore, the final ranking of alternatives can be given according to:

$x_5 \succ x_4 \succ x_2 \succ x_1 \succ x_6 \succ x_3$.

It can be concluded that x5, x4 and x1 are the top-3 courses that need to be optimized in the next grade.

5.2 Comparisons with existed decision procedures

By using the optimization model with equal weights, i.e., Eq. (7) provided by Wan and Dong [29], the results are

$x_5 \succ x_4 \succ x_6 \succ x_1 \succ x_2 \succ x_3$.

The top-2 candidates are the same, while the third course changes to be x6.

Next, the results would be compared with the aggregation operator-based decision procedure given by [33].

By using the obtained weighting vector of three 'expert' and setting equal weights of all attributes, the aggregated expert decision information via basic uncertain linguistic information ordered weighting operator (BULIOWA) can be shown in the following Table 7.

Table 7
Aggregated group decision information

Agg.	*u*1	*u*2	*u*3	*u*4	*u*5
*x*1	$(\Delta(3.1162);.7040)$	$(\Delta(2.9954);.7183)$	$(\Delta(3.2485);.6932)$	$(\Delta(3.1954);.6563)$	$(\Delta(3.3208);.6687)$
*x*2	$(\Delta(3.3300);.6472)$	$(\Delta(3.2248);.7159)$	$(\Delta(3.2241);.7235)$	$(\Delta(3.0561);.6279)$	$(\Delta(3.2521);.7035)$
*x*3	$(\Delta(3.0710);.7012)$	$(\Delta(3.1496);.7447)$	$(\Delta(3.0542);.6799)$	$(\Delta(3.3273);.6404)$	$(\Delta(3.0914);.6527)$
*x*4	$(\Delta(3.1006);.6386)$	$(\Delta(3.3096);.7450)$	$(\Delta(3.0261);.6378)$	$(\Delta(3.3700);.6468)$	$(\Delta(3.4567);.7347)$
*x*5	$(\Delta(3.2908);.7109)$	$(\Delta(3.1033);.7154)$	$(\Delta(3.3746);0.6726)$	$(\Delta(3.4162);.7007)$	$(\Delta(3.2895);.7079)$
*x*6	$(\Delta(2.9000);.6757)$	$(\Delta(3.1561);.6747)$	$(\Delta(3.2221);.6742)$	$(\Delta(3.3607);.7079)$	$(\Delta(3.0666);.6650)$

In Table 7, the basic uncertain linguistic information is the aggregation of corresponding values located at the same position of three decision matrices, which is realized according to

$$\text{BULIOWA}\left(b_1, b_2, \ldots, b_n\right) = \sum_{i=1}^{n} \hat{w}_i b_{\sigma(i)} = \left(\Delta\left(\sum_{i=1}^{n} \hat{w}_i \psi_{\sigma(i)} \right); \sum_{i=1}^{n} \hat{w}_i c_{\sigma(i)} \right),$$

where $b_{\sigma(i)} = \left(\Delta\left(\psi_{\sigma(i)}\right); c_{\sigma(i)}\right)$ is the i-th largest of $b_i = \left(\Delta\left(\psi_i\right); c_i\right)$, and $\hat{w} = \left(w_1, \cdots, w_n\right)$ is the associated weighting vector.

Next, since the weights associated to all attributes are the same, the final aggregation of each candidate can be also derived by using BULIOWA operator, and

Agg(x1)= $(\Delta(3.1753);.6881)$, Agg(x2)= $(\Delta(3.2174);.6926)$, Agg(x3)= $(\Delta(3.1387);.6838)$, Agg(x4)=

$(\Delta(3.2526);.6806)$, Agg(x5)= $(\Delta(3.2949);.7015)$, Agg(x6)= $(\Delta(3.1411);.6795)$.

Then, according to the comparison law, we have $x_5 \succ x_2 \succ x_4 \succ x_1 \succ x_3 \succ x_6$.

The top-3 courses are the same as the results produced by our developed method. On the other hand, the order relation between x2 and x4 is contrary.

By comparing the results produced by three models, it can be found that our result is like another two results. As a result, better adaptability and generality.

6. Discussions

According to the comparisons shown in subsection 5.2, the developed decision procedure performs better adaptability and generality. In this subsection, the strengths and weaknesses of the developed model would be summarized.

Firstly, like another soft sets-based decision-making procedure, the developed decision method can handle uncertainties from the structural perspective and the cognitive level. Besides, it can dynamically reflect the superiority relationship between objects through the advantage matrix, thus improving parameter reduction and decision-making efficiency. Next, since basic uncertain linguistic information can reflect both the qualitative assessment and its credibility degree, the unity of quantitative and qualitative decision-making can be realized.

On the other hand, compared to the aggregation operator-based decision method, the computational complexity of the model would be relatively high. Besides, as a general limitation of decision models based on soft sets, its dynamic adaptability is insufficient. In other words, such models are more feasible to handle static data.

7. Conclusions and future remarks

In many practical applications, decision makers are faced with uncertainties that come from incomplete information or unknown knowledge. Chen et al. [14] introduced the concept of basic uncertain linguistic information to describe such uncertainty contained both in the subjective linguistic evaluation and certainty level. What's more, the notion of soft set theory has proved to be a useful and powerful tool to deal with uncertainty in decision-making. In this paper, we discussed mathematical basis of basic uncertain linguistic information soft sets including set operations and similarity measures. As an application of the novel soft set, a multi-criteria group decision algorithm by using traditional decision procedure of soft set theory is given.

In the future, the algebra structure of basic uncertain linguistic information soft sets can be further considered. Information measures include distance measures and entropy measures corresponding to such fuzzy soft sets can also be studied. Furthermore, one can also extend the applications of basic uncertain linguistic information soft sets. For instance, the application of basic uncertain linguistic information soft sets in theoretical forecasting problems and in practical medical diagnosis problems.

Author Contributions

Conceptualization, Peitao Qin and Dragan Pamucar; methodology, Zhifu Tao and Dragan Pamucar; software, Zhifu Tao; validation, Peitao Qin and Dragan Pamucar; formal analysis, Zhifu Tao; writing—original draft preparation, Zhifu Tao; writing—review and editing, Peitao Qin and Dragan Pamucar. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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