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Government Subsidies and Patent Race Strategies: A BI-Objective Portfolio Optimization Model for Communication Technology Innovation

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ABSTRACT

This study examines the strategic challenges encountered by communication enterprises engaged in government-subsidised patent competitions, where the effective distribution of R&D resources is essential to offset competitive disadvantages. To address this issue, a bi-objective portfolio optimisation model is proposed, aiming to maximise net returns while simultaneously reducing semi-absolute deviation risk, with consideration of substitution costs and policy-driven subsidies. Using MATLAB-based numerical simulations, the analysis demonstrates that firms with weaker competitive positions can improve their performance in patent races by reallocating investments across significant technological areas in line with their varying levels of risk tolerance. The findings further suggest that when firms possess greater autonomy in decision-making, concentrating resources in core self-interest technologies enhances their chances of achieving success in patent races. Overall, the research contributes theoretical and practical perspectives on R&D portfolio management within policy-supported incentive frameworks.

1. Introduction

Since the beginning of the 21st century, the government's growing emphasis on supporting scientific and technological innovation, particularly through subsidy schemes in communication technologies, has created both opportunities and challenges for enterprises engaged in patent races [1-4]. Drawing on the stages outlined in the technology life cycle theory, the introduction phase highlights internal capacity building, the growth phase emphasizes responses to competitive forces, and the maturity phase depends heavily on policy interventions. Within the maturity stage, the notion of leveraging policy resources becomes especially critical. Subsidies serve as a vital external mechanism that can ease enterprises' R&D funding burdens, diminish investment risks, and stimulate participation in high-risk but potentially lucrative technological innovation projects [5]. Nevertheless, the challenge for firms lies in how to strategically allocate resources and optimise their investment portfolios to secure maximum returns while controlling risks.

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Government subsidies extend beyond their role as an economic stimulus; they represent an essential instrument for enhancing resource allocation and fostering innovation [6]. By reducing the likelihood of innovation-related failures, they provide crucial support for projects with considerable risks but equally high potential returns. Such support may take the form of direct measures, including financial subsidies, preferential tax policies, or low-interest loans, as well as indirect initiatives, such as creating dedicated funds that encourage industry-university-research collaboration and transform scientific outcomes into practical outputs. Moreover, governments can shape favorable innovative environments by implementing policies and regulations that streamline approval processes, strengthen intellectual property protection, and promote fair market competition.

From the standpoint of National Innovation Systems (NIS), subsidies play a much broader role than simple financial support. They are strategic interventions that reinforce the innovation ecosystem by fostering collaboration across industries, universities, and research institutions, enabling knowledge diffusion, and lowering competitive barriers for firms in less advantageous positions. While this paper employs a quantitative model to evaluate firm-level strategies, it situates the analysis within this larger policy and innovation framework. It argues that a clear understanding of the mechanisms through which subsidies shape corporate R&D portfolio strategies is essential for the development of national-level policies aimed at strengthening technological leadership and competitiveness in critical sectors such as communication technology. This research explores how telecommunications enterprises participate in patent competitions under government funding support, concentrating on optimising R&D portfolio strategies that incorporate subsidies, expected returns, risk measures, and substitution costs. The objective is to establish a bi-objective optimisation framework that seeks to maximise net returns while minimising investment risks for late entrants. The study begins by identifying research gaps and framing key questions, then specifying major variables, including investment allocations across technological stages, projected outcomes, and benchmarks for venture risk, within the context of communication technology. A mathematical framework is subsequently constructed to integrate these variables and provide guidance for investment strategies of firms attempting to catch up in patent races.

To validate the robustness and practical value of this model, MATLAB-based simulations are carried out to examine optimal portfolio choices under various risk preferences and to evaluate their impact on enterprise performance. The findings demonstrate that government subsidies substantially enhance the financial resilience of latecomer firms, enabling them to adapt their portfolios with greater flexibility and thereby improve overall competitiveness. Furthermore, empirical case data confirm both the validity and the applicability of the proposed framework. Overall, the results indicate that subsidies are not merely fiscal stimulus but serve as an important policy instrument for advancing technological innovation, improving resource allocation, and reducing the risks inherent in high-return innovation projects.

2. Literature Review

2.1 Patent Race

The dynamics of global patent competition have shifted from an emphasis on sheer patent numbers to a focus on the strategic value and industrial significance of patents. This transformation reflects a broader movement from prioritizing quantity to valuing quality, from isolated patent conflicts to integrated portfolio management, and from spontaneous market-driven rivalry to deliberately structured innovation contests. At present, patent races are centered on the strategic accumulation and deployment of patent rights, which serve as key instruments of competitive positioning. In advanced technological sectors, both at national and organisational levels, the acquisition and management of patents are critical to securing leadership. The success of enterprises in these contests is increasingly tied to the strategic governance of intellectual property resources and their organisational capacities [7-9].

According to Al-Fazari and Teng [10], participants compete not only to dominate markets but also to collectively raise the technological value of entire industries. Their analysis demonstrates that innovative efforts peak when rivals show comparable R&D cost efficiency, with economies of scale reinforcing competition among equally matched firms but diminishing activity when disparities are substantial. Fama and French [11] provided empirical evidence of such competitive dynamics, particularly in the field of information technology, and observed that leading firms generate more follow-up innovations, with subsequent research strongly aligned to earlier patented inventions. Ren [12] explored the application of the Verdmog model in explaining China's technological innovation patterns, while [12] highlighted the growing integration of computing and communication within artificial intelligence, which enhances data processing and organisation and accelerates deep learning progress. These trends underscore the strategic significance of modern computer communication technologies and the increasing corporate focus on advancing them.

Patent race theory has also been structured into a two-stage model based on a single-interval dynamic game, in which the innovation process begins with fundamental research and progresses into proprietary development, culminating in patent grants. The Verdmog model, a widely applied framework, captures these dynamics and incorporates first-mover advantages. Building on this foundation, Gilbert and Newbery [13] analysed the stage-dependent nature of competition, while later contributions expanded the model to optimise patent races. For example, Merges [14] noted that strong early-stage protection can restrict the entry opportunities of later competitors, whereas [15] integrated time into the model to examine forward protection mechanisms. Feng et al. [16] advanced the debate by showing that subsequent competition arises once initial outcomes are disseminated, and that dominant firms systematically commit more resources. Similarly, Kim and Koo [17] proposed incorporating compulsory contingent licensing into patent systems to reduce duplication of R&D efforts. Other influential studies include Wagner [18] on early-stage innovation prospects, [19] on informal IP protection within growth frameworks, and [20] on strategic patenting under uncertainty. More recent analyses by Jensen and Showalter [21] stress the importance of high-quality patent portfolios for consolidating market advantage and fostering innovation across developmental phases.

As firms increasingly adopt patents for strategic purposes, motivations have expanded beyond safeguarding intellectual property to support broader organisational objectives. Modern patent strategies have thus evolved into comprehensive management systems that seek to maximise the long-term value of patent portfolios. This involves not only acquiring and administering patents but also coordinating related processes such as development, filing, maintenance, licensing, and enforcement. Effective integration of these functions has become an essential capability for enterprises competing in global high-technology industries. Consequently, patent management has transitioned from a supporting legal role to a central strategic function that combines traditional IP operations with more assertive approaches, including litigation strategies, patent pooling, and collaborative portfolio optimisation. Wang et al. [9] recommend further systematisation of patent analysis and decision-making within competitive contexts, calling for closer alignment of strategic IP activities with corporate innovation pathways. Likewise, Yang and Fan [22] link the granting of higher-quality patents in emerging strategic industries with greater technological sophistication, deeper levels of protection, and stronger competitive advantage.

2.2 Portfolio Risks in Patent Race

Academic inquiry into patent-related investment began in the 1970s, when scholars first started examining R&D investment strategies. As global markets became increasingly complex, economic uncertainty was gradually incorporated into research analyses. Patent investment is interpreted in both narrow and broad contexts, encompassing short- and long-term objectives, as well as tangible and intangible outcomes, each associated with varying degrees of profitability. Differences in intellectual property awareness and policy frameworks across nations have also contributed to divergent domestic investment practices and risk exposures. Investors are therefore required to select appropriate models tailored to their specific contexts and, in some cases, engage in joint funding or coordinated partnerships to mitigate uncertainties. Certain investment models linked to patents face significant regulatory and policy constraints. For example, newly registered patents often involve lengthy approval procedures, and unsuccessful applications cannot be capitalized through transfers, making documentation and compliance with regulatory review critical. Moreover, some investment strategies rely heavily on local subsidies, which, if modified or withdrawn, may expose firms to substantial financial losses. Subsidies should not be viewed solely as direct financial assistance to enterprises, but as a broader policy instrument aimed at shaping national innovation ecosystems. They serve to correct market inefficiencies, reduce the risks inherent in fundamental research, and enhance the international competitiveness of national technologies. Nonetheless, environmentally oriented investments supported through subsidies and inter-firm collaboration have at times resulted in disputes or litigation despite initial government backing.

Several scholars have incorporated risk preferences into quantitative models to examine their influence on corporate investment decisions. Risk preference, defined as the psychological orientation of decision-makers toward uncertainty, highlights differences in judgement and perception. Consequently, even under identical investment conditions, firms may pursue different strategies due to variations in risk appetite, resulting in distinct strategic outcomes. Previous studies on patent-related risks have largely centred on how risk factors affect decision variables such as portfolio structure, yet they have rarely considered the role of competing entities' risk preferences within patent race contexts. Garleanu et al. [23] identified a strong "displacement effect" generated by innovation, while [24] created a technical risk index to explain volatility in French portfolio returns. Banerjee and Sengupta [25] observed that industry leaders in innovation often face lower systematic risks compared to followers and proposed a winner-takes-all model to examine how strategic rivalries influence both risk exposure and expected returns.

Further contributions in this area include Lee et al. [26], who investigated how firm-level risk preferences affect optimal R&D strategies and portfolio allocations under diverse market conditions and risk scenarios. Similarly, Beauchêne [27] analysed duopolistic competition in R&D and product markets, finding that ambiguity aversion intensifies perceived rivalry, while risk tolerance diminishes incentives for R&D spending. Yuan [28] stressed the interdependence of firms during patent contests, observing that investment decisions are often mutually responsive. Beyond internal innovation efforts, dominant firms may seek incremental investments in, or acquisitions of, weaker competitors to maximize returns. However, early success in a patent race does not guarantee sustained advantage, as competitive positions evolve in response to each firm's strategies. Yuan also underlined the pervasive uncertainty in patent races, suggesting that firms must continually assess their competitive positions. Despite the complexity of projects and unpredictable dynamics, enterprises can adapt their R&D investment strategies by interpreting available information and aligning actions with their strategic stance. In parallel, Li and Wu [29] argued that in China, where patent applications have rapidly expanded, regulatory authorities should not only strengthen quality oversight but also provide practical guidance on patent strategy and intellectual property management, particularly for small and medium-sized enterprises.

2.3 Research Gap and Contribution

A review of the literature reveals that patent competition has emerged as a central focus for intellectual property research. Nonetheless, much of the existing scholarship concentrates on cases where firms compete on relatively equal footing, overlooking the continued participation of less advanced enterprises in high-potential technological fields. This is particularly evident in the communications sector, where firms often remain active despite repeated R&D setbacks. In the context of ongoing transformations in communication network technologies and a rapidly changing market environment, this study introduces a more practical bi-objective portfolio framework that explicitly accounts for investment risks associated with communication technologies. The research highlights the strategic challenges faced by lagging firms in patent races and proposes a dual-objective optimization model designed to maximize net returns while minimizing semi-absolute deviation risk. Through an analysis of model characteristics and numerical sensitivity testing of core parameters, the study provides investment guidance for firms seeking to mitigate competitive disadvantages. The findings indicate that carefully calibrated R&D investment choices in later phases can generate successful innovation outcomes, even when earlier strategic decisions were less effective.

Viewed from the perspective of NIS, this work explores how firm-level responses to government subsidies represent critical micro-level mechanisms embedded within broader institutional structures. Subsidies are interpreted here as direct policy tools intended to reinforce the NIS by enhancing the innovative capacity of individual enterprises. Finally, this study contributes to ongoing debates in the knowledge economy by quantifying the investment decisions underlying patent-driven knowledge production. It reinforces the view that innovation policy functions as economic policy, and that effective allocation of resources at the firm level is essential for sustaining competitiveness in technology-intensive industries.

3. Problem Description and Model Formulation

3.1 Status of Government-Subsidized R&D Investment by Enterprises in the Patent Race

Within the context of an increasingly globalised economy and the accelerated advancement of science and technology, communication enterprises are confronted with intensifying international competition. This environment compels corporate decision-makers to prioritise the cultivation of sustainable competitive advantages through technological innovation, particularly by securing advantageous positions in patent races. Patent competition extends beyond the immediate financial returns of firms, serving instead as a decisive factor in shaping long-term market standing. When a pioneering technology achieves patent protection, the initial innovator is able to secure monopoly rents, whereas late entrants are often excluded from deriving significant benefits. Consequently, firms must adopt carefully calibrated strategies and optimise their investment portfolios to strengthen their prospects in patent-driven competition.

Since the beginning of the twenty-first century, heightened governmental commitment to fostering scientific and technological innovation, most notably through subsidy programmes in the communications sector—has introduced both opportunities and challenges for firms engaging in patent competition. As a critical external policy instrument, subsidies can alleviate constraints on R&D funding, reduce investment uncertainty, and encourage enterprises to pursue high-risk yet high-potential innovation endeavours. Nonetheless, the simultaneous need to allocate resources prudently and to optimise investment portfolios to maximise expected returns while limiting exposure to risk presents an urgent strategic dilemma for firms seeking to capitalise on these incentives. Within this competitive environment, the formulation of effective R&D portfolio strategies that enable late-moving firms to overcome structural disadvantages is of considerable significance. The principal contributions of this chapter are summarised as follows:

1. By combining the technological context of mobile communication networks with theories of patent races and portfolio investment, this study constructs a mathematical model to simulate the R&D investment decisions of lagging firms under conditions of government support. The model is designed to pursue dual objectives, namely maximising expected returns and minimising investment risk.
2. Through computational simulations, optimal portfolio allocation strategies for late entrants are identified across varying degrees of risk exposure. The experiments demonstrate how resources can be distributed across core technology domains, while also examining the extent to which government subsidies influence investment returns under different levels of risk.

3.2 Venture Portfolio Model with Government Subsidies Under Patent Race

3.2.1 Problem Description

This study examines the function of government subsidies in shaping patent competition within the telecommunications sector, where firms strategically allocate resources to advance communication technologies. Investment in such technological domains is inherently uncertain and characterised as risk-bearing, yet these ventures are fundamental drivers of innovation. They enable the transformation of advanced technological achievements into practical applications, thereby fostering industrial upgrading and strengthening the competitive position of firms. At the same time, public R&D subsidies serve as an external financial mechanism that directly supports patent-driven activities. Under conditions of liquidity constraints, and after considering the costs associated with entry and withdrawal across different asset categories, firms are modelled as pursuing dual objectives: the maximisation of net investment returns and the minimisation of expected risk exposure. Furthermore, once a leading enterprise achieves success in the initial stages of research, this accomplishment can stimulate rival firms to intensify their innovation efforts. By committing greater resources to accelerate progress through the current stage, these late entrants enhance their ability to transition into subsequent phases of technological development.

3.2.2 Model Assumptions

Table 1
Description of Symbols

Decision Variables	Definition
x_{it}	t Percentage of investment in technological fields during the period i , $i = 1,2,...,n$ and $t = 1,2,...,T$
Parameter	**Definition**
r_{it}	t Expected return on technology area in the period
t	Patent investment (R&D) period, the $t = 1,2,...,T$
α_i	Percentage of investments in i technologies by latecomer firms
w_{it}	t Period i Risk of Venture Investments, the $i = 1,2,...,n$
β_i	i Risk benchmarks for venture capital investments. $i = 1,2,...,n$
k_i	i Replacement (input or exit) costs in the technology area, the $i = 1,2,...,n$
u_{it}	t Period i Upper limit of investment ratio in technology, $i = 1,2,...,n$
λ_i	i Percentage of government subsidies in the technology sector, period $i = 1,2,...,n$

Based on the foregoing problem formulation, the decision variables and parameters applied in this study are presented in Table 1. Although the empirical setting and numerical simulations in this study are situated within the communication technology industry, the proposed bi-objective optimisation framework possesses broader applicability. Its central constructs—expected return (R), investment risk (θ), replacement cost (C), and subsidy rate (γ)—represent fundamental determinants of R&D investment behaviour across technology-intensive sectors engaged in patent races. The framework is not constrained by industry-specific characteristics; rather, it can be extended to any domain where innovation unfolds sequentially under uncertainty, shaped simultaneously by strategic rivalry and policy intervention.

3.2.3 Model Assumptions

In the course of R&D investment targeting critical technologies, withdrawal is a common outcome given prevailing uncertainties. The financial and temporal losses incurred from such withdrawal are collectively referred to as exit costs. Within the dynamics of patent competition, the aggregated expenditure associated with exit costs can be expressed as:

$$K=\sum_{i=1}^{n}k_i\left|\sum_{t=1}^{T}x_{it}-\alpha_i\right|.$$

The anticipated return generated from the portfolio of assets allocated by late-entrant firms to each critical technology domain is represented as:

$$R=\sum_{i=1}^{n}\sum_{t=1}^{T}r_{it}x_{it},$$

Where, $\left|\sum_{t=1}^{T}x_{it}-\alpha_i\right|$ denotes the difference between the total investment ratio in the technology area and the investment ratio in the three periods.

The return on investment in communication technology R&D is defined as the net gain, calculated as the sum of profits and related income from such investments minus the associated losses, expressed as R−K. The government subsidy is denoted as:

$$S=\sum_{i=1}^{n}\lambda_i\sum_{t=1}^{T}r_{it}x_{it}.$$

Therefore, the net return function of the portfolio of the latecomer firms is as follows:

$$f(x)=R-K+S=\sum_{i=1}^{n}((1+\lambda_i)\sum_{t=1}^{T}r_{it}x_{it}-k_i\left|\sum_{t=1}^{T}x_{it}-\alpha_i\right|).$$

For the portfolio $x_i=(x_{i1},x_{i2},...,x_{it})'$, its semi-absolute deviation risk function can be expressed as:

$$g(x)=\sum_{t=1}^{T}\left|\min\left\{0,\sum_{i=1}^{n}(w_{it}-\beta_i)x_{it}\right\}\right|.$$

The semi-absolute deviation risk function is incorporated into portfolio theory to address the optimisation challenge of determining the proportional allocation of investments. The specific dual-objective programming model (P_0) can be constructed as follows:

$$(P_{3-0})\quad \max f(x),\ \min g(x)$$

$$\text{s.t.}\ \sum_{t=1}^{T}\sum_{i=1}^{n}x_{it}=1,\quad 0\le x_{it}\le u_{it}.$$

After adding government subsidies, conservative firms have a minimum standard for investment returns f_0 , then the original bi-objective model (P_0) is transformed into the following single-objective model (P_1).

$$(P_{3-1}) \quad \min g(x) = \sum_{t=1}^{T} \left| \min\left\{0, \sum_{i=1}^{n} (w_{it} - \beta_i) x_{it} \right\} \right|$$

$$\text{s.t.} \sum_{t=1}^{T} \sum_{i=1}^{n} x_{it} = 1, \quad (1)$$

$$0 \le x_{it} \le u_{it}, \quad (2)$$

$$f(x) = \sum_{i=1}^{n} ((1+\lambda_i) \sum_{t=1}^{T} r_{it} x_{it} - k_i \left| \sum_{t=1}^{T} x_{it} - \alpha_i \right|) \ge f_0. \quad (3)$$

Thus, introduce new variables that satisfy the following conditions z

$$\sum_{i=1}^{n} k_i \left| \sum_{t=1}^{T} x_{it} - \alpha_i \right| \le z, \quad (4)$$

Accordingly, constraint (3) can be reformulated as:

$$\sum_{i=1}^{n} (1+\lambda_i) \sum_{t=1}^{T} r_{it} x_{it} - z \ge f_0. \quad (5)$$

The single-objective model (P_1) can be transformed into:

$$(P_2) \min g(x) = \sum_{t=1}^{T} \left| \min\left\{0, \sum_{i=1}^{n} (w_{it} - \beta_i) x_{it} \right\} \right|$$

s.t. $(1),(2),(4),(5).$

In addition, the ascending order method is utilized to transform the model (P_1), specifically by introducing the new variables e_i^+ and e_i^-. e_i^+ The equivalent transformations of and e_i^- as new variables.

$$e_i^+ = \frac{\left| \sum_{t=1}^{T} x_{it} - \alpha_i \right| + (\sum_{t=1}^{T} x_{it} - \alpha_i)}{2}, \quad e_i^- = \frac{\left| \sum_{t=1}^{T} x_{it} - \alpha_i \right| - (\sum_{t=1}^{T} x_{it} - \alpha_i)}{2}.$$

The set of constraints (4) can thus be equivalently reformulated as:

$$\begin{cases} \sum_{i=1}^{n} k_i (e_i^+ + e_i^-) \le z, & (6) \\ e_i^+ - e_i^- = \sum_{t=1}^{T} x_{it} - \alpha_i, & (7) \\ e_i^+ \cdot e_i^- = 0, & (8) \\ e_i^+ \ge 0,\ e_i^- \ge 0. & (9) \end{cases}$$

With respect to the objective function specified in the preceding single-objective model,

$$\left| \min\left\{0, \sum_{j=1}^{n} (w_{it} - \beta_i) x_{it} \right\} \right| = \max\left\{0, -\sum_{j=1}^{n} (w_{it} - \beta_i) x_{it} \right\} = \frac{\left| \sum_{i=1}^{n} (w_{it} - \beta_i) x_{it} \right| - \sum_{i=1}^{n} (w_{it} - \beta_i) x_{it}}{2}.$$

In a similar manner, the ascending order method is employed to incorporate newly defined variables l_i^+, l_i^-

$$l_t^+ = \frac{\left| \sum_{i=1}^{n} (w_{it} - \beta_i) x_{it} \right| + \sum_{i=1}^{n} (w_{it} - \beta_i) x_{it}}{2}, \quad l_t^- = \frac{\left| \sum_{i=1}^{n} (w_{it} - \beta_i) x_{it} \right| - \sum_{i=1}^{n} (w_{it} - \beta_i) x_{it}}{2}.$$

On the basis of the preceding equations, the formulation can be equivalently expressed as the following system:

$$\begin{cases} l_t^+ - l_t^- = \sum_{i=1}^{n} (w_{it} - \beta_i) x_{it}, & (10) \\ l_t^+ \cdot l_t^- = 0, & (11) \\ l_t^+ \geq 0, & (12) \\ l_t^- \geq 0. & (13) \end{cases}$$

This study employs the principle of equivalent transformation as a key methodological approach. Given the inherent complexity of the original model, direct solutions are often intractable. Therefore, the problem is reformulated through equivalence, enabling the conversion of an unsolved formulation into one that can be addressed using established analytical techniques. Through this strategy, the model becomes more amenable to systematic investigation while preserving its essential characteristics. Accordingly, the original formulation is restructured into the following equivalent model:

$$(P_3) \quad \min g(x) = \sum_{t=1}^{T} l_t^-$$

s.t. (5.1), (5.2), (5.5)-(5.13).

(P_3) The objective function $g(x)$ in the model is the semi-absolute deviation risk function of the portfolio of latecomer firms after the model transformation. Constraint (1) represents the budgetary condition governing enterprise investment in communication technologies, requiring that the cumulative proportion of allocations across all key technologies equals one. Constraint (2) stipulates both upper and lower bounds on the investment ratio, where the maximum permissible share for a given technology in phase is denoted as , and the minimum bound is fixed at zero. Constraint (5) introduces the requirement that investment decisions must satisfy a minimum expected return, ensuring that outcomes remain above a threshold regarded as acceptable to investors. Constraint corresponds to the reformulated version of equation (4), imposing an upper limit on the aggregate expenditure associated with replacement costs of assets invested by latecomer firms across various technology areas. Constraints (6)–(9) represent the transformed conditions that address the absolute value component of the objective function. To handle the complementary constraints (8) and (11), the large-M approach is employed, enabling their conversion into the following equivalent set of constraints:

$$\begin{cases} e_i^+ \leq M(1 - v_{e_i}) \\ e_i^- \leq M v_{e_i} \end{cases} \text{ and } \begin{cases} l_t^+ \leq M\left(1 - v_{l_t}\right) \\ l_t^- \leq M v_{l_t} \end{cases}$$

Where $v_{e_i}, v_{l_t}, \mathrm{M} = +\inf$. In short, the economic meaning of the model (P_3) refers to how the latecomer firms allocate various assets to invest in key communication technologies to minimize the risk of semi-absolute deviation of their portfolios and maximize the net return of their portfolios under the premise of satisfying the above constraints.

4. Numerical Experiments and Results Analysis

This study examines the optimal strategic approaches that late-entering firms should pursue while still positioned in the initial stage of research. Building on the six critical technological domains outlined in the preceding chapter—large-scale antennas, multi-connectivity solutions, distributed cloud frameworks, flexible sub-frame scheduling, next-generation airport systems, and emerging spectrum resources—this section analyses the composition of an optimal investment portfolio once government subsidies are incorporated into the decision-making process.

4.1 Model Calculation

Whereas the preceding section developed a theoretical model to characterise the net portfolio returns of late-entrant firms, the underlying parametric relationships were expressed in functional form. Consequently, this section applies numerical simulations to explore the dynamic interplay among decision variables, the objective function, and associated parameters. Through this process, an optimal allocation strategy across specific communication technology domains is derived. Acquiring firm-level, fine-grained experimental data presents considerable practical challenges, which constitutes a principal rationale for employing simulation methods in this study. To enhance the reliability and authenticity of the simulation inputs, extensive publicly available information was collected and systematised from online sources, including industry reports and technical documentation.

In addition, practitioners and experts in the communications sector were consulted to obtain empirical insights and parameter value ranges. During the preliminary phase of experimental design, reports from authoritative market research institutions, such as AiMedia Consulting, were also incorporated as reference materials. Drawing on these combined sources, data ranges and distributions were established to capture the representative characteristics of the research context. The model was subsequently implemented and solved using MATLAB. Table 2 presents the parameter settings, which reflect, among others, the finding of Zhao Hua et al. that increasing R&D subsidies does not always translate into higher corporate profits, and that a subsidy rate of approximately 0.5 is particularly effective in enhancing profitability.

Table 2
Model Parameter Values

Symbol	Numerical Value
r_{i1}	300.190300180 .120 .350 (Million US Dollars)
r_{i2}	180180.200100 .280 .300 (Million US Dollars)
r_{i3}	280.250.250.190.120.30 (Million US Dollars)
β_i	*i* Risk Benchmark in the Field of Technology, 50200500200150300
k_i	*i* The Cost of Removal (Investment or Exit) in the Field of i Technology is 53611.8 Million US Dollars
λ_i	*i* The Cost of Removal (Investment or Exit) in the Field of i Technology is 53611.8 Million US Dollars

4.2 Optimal Portfolio Strategies at Different Risk Levels

Firms engaged in frontier communication technologies often adopt divergent monetisation logics. Some emphasise the direct commercial exploitation of proprietary assets, while others generate value by transferring technological outputs and recycling the proceeds into subsequent R&D cycles and adjacent innovation ventures. Despite these strategic differences, a common thread lies in the centrality of investment risk evaluation. Risk metrics not only guide decisions on sustaining or withdrawing from particular technology domains but also underpin the overarching imperative of identifying portfolio configurations capable of sustaining long-term competitive relevance within patent races. Once government subsidies are incorporated into the decision calculus, and assuming that investors in follower firms impose a minimum acceptable net return while displaying heterogeneous risk appetites, the resulting equilibrium allocation of R&D resources emerges as depicted in Figure 1. This outcome reflects stochastic risk assignments across the six designated technology domains and is generated using the methodological approach formalised in the preceding section.

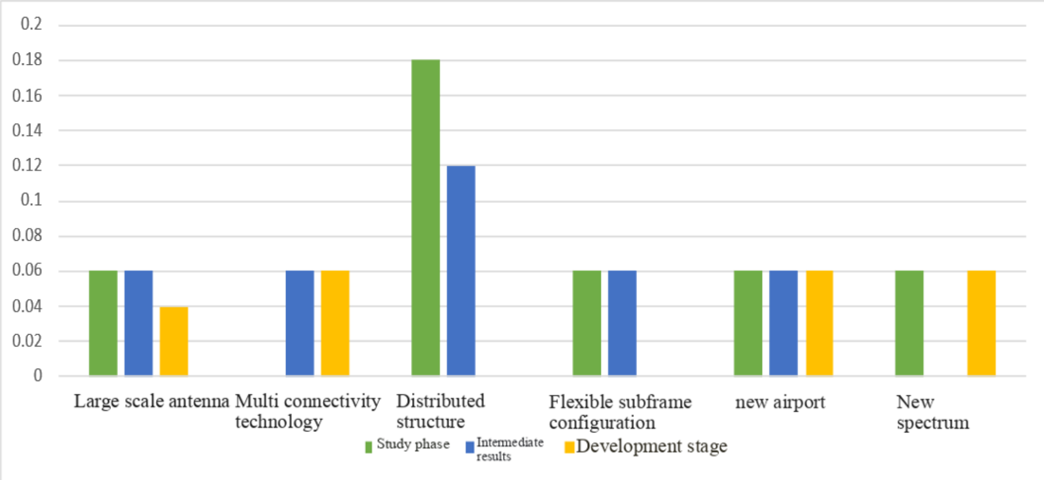

Fig. 1: Optimal Investment Portfolio Strategy in 5G Communication Technology Field When w_{it} is Randomized

Figure 2 illustrates the distribution of investment across six core 5G technologies at successive development stages following the incorporation of government subsidies. The horizontal axis denotes the specific technology domains, while the vertical axis reflects corresponding investment ratios. Distinct colours are used to differentiate the research, intermediate results, and development stages. The simulation results reveal several notable patterns. Large-scale antenna technology commands sustained emphasis during the research and intermediate phases, with relatively diminished investment in later development.

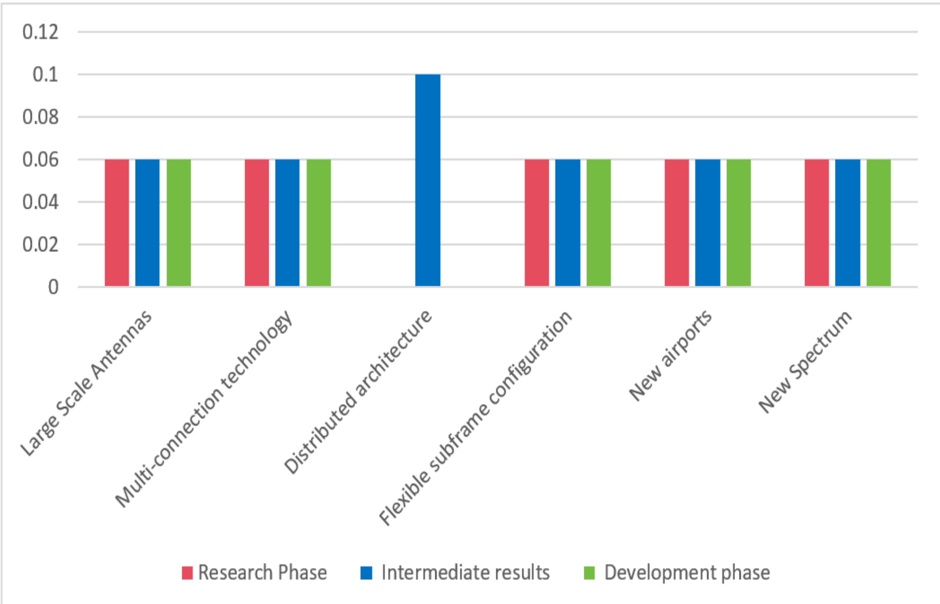

Fig. 2: Optimal Portfolio Strategy in 5G Communication Technologies When $w_{it} = 100$

Multi-connectivity technologies attract greater allocation in the intermediate and development stages. Distributed cloud architecture consistently exhibits the highest investment ratio across all technologies, peaking in the research phase and maintaining a disproportionately large share at the intermediate stage. Flexible Subframe Configuration is excluded from development-stage investment under subsidised conditions, thereby disappearing from the final portfolio. By contrast, New Airports sustains a steady allocation across all three phases, while New Spectrum shows greater investment concentration during both the research and development stages.

The results indicate that when w_{it} attains higher values, the portfolio allocations across 5G technologies exhibit notable adjustments. Large-scale antennas shift predominantly toward the research and intermediate stages. Multi-connectivity technologies are reoriented toward research and intermediate outcomes, while distributed cloud architecture continues to hold the highest investment share, concentrated mainly in the research phase.

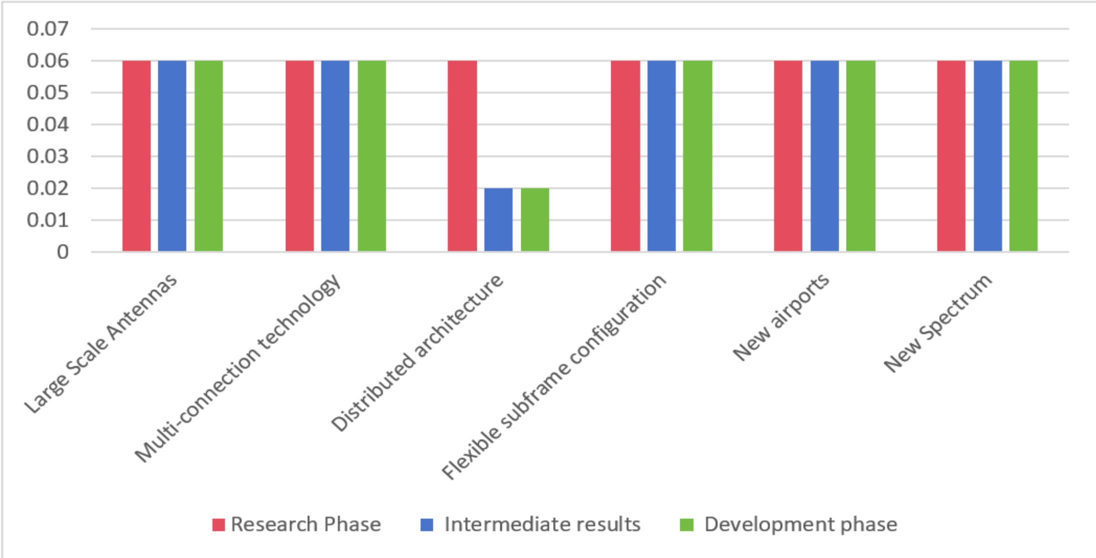

Fig. 3: When $w_{it} = 200$, Optimal Portfolio Strategy in 5G Communication Technology Sector

Flexible Subframe Configuration undergoes a reversal from its earlier allocation, becoming directed toward the development stage. Investment patterns for New Airports remain broadly consistent with the original results, with no allocation to intermediate outcomes. By contrast, New Spectrum is withdrawn from the development stage, achieving a more balanced distribution between the research and intermediate phases. These dynamics are depicted in Figures 3 and 4.

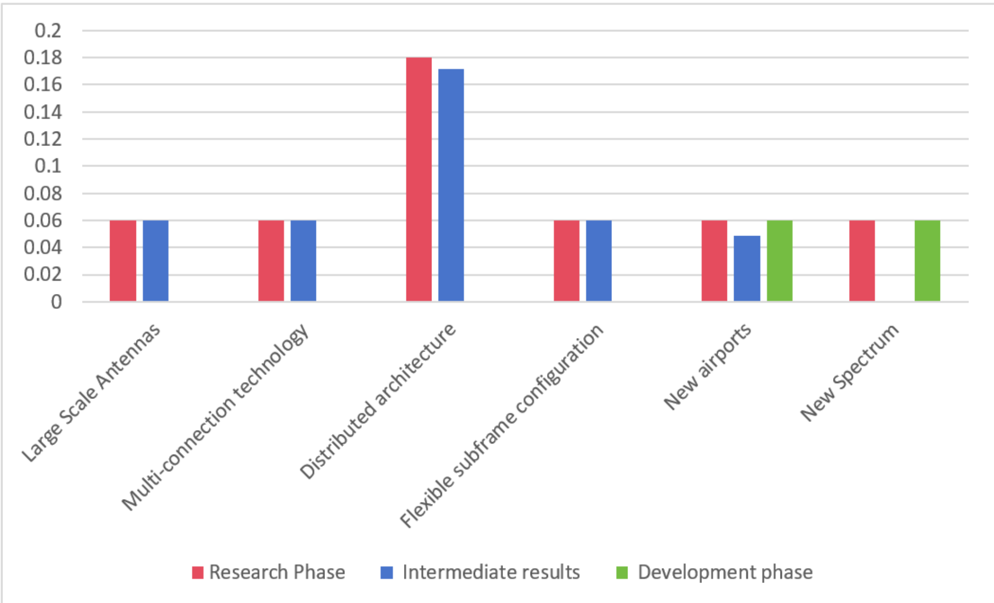

Fig. 4: When $w_{it} = 300$, Optimal Portfolio Strategy in the Field of 5G Communication Technologies

When the latecomer investors have minimum requirements for net returns and high investment risks (i.e., when the risks of each key technology field are $w_{it} \geq 400$ in all periods), the optimal investment portfolio of latecomer enterprises is always shown in Figures 5 and 6.

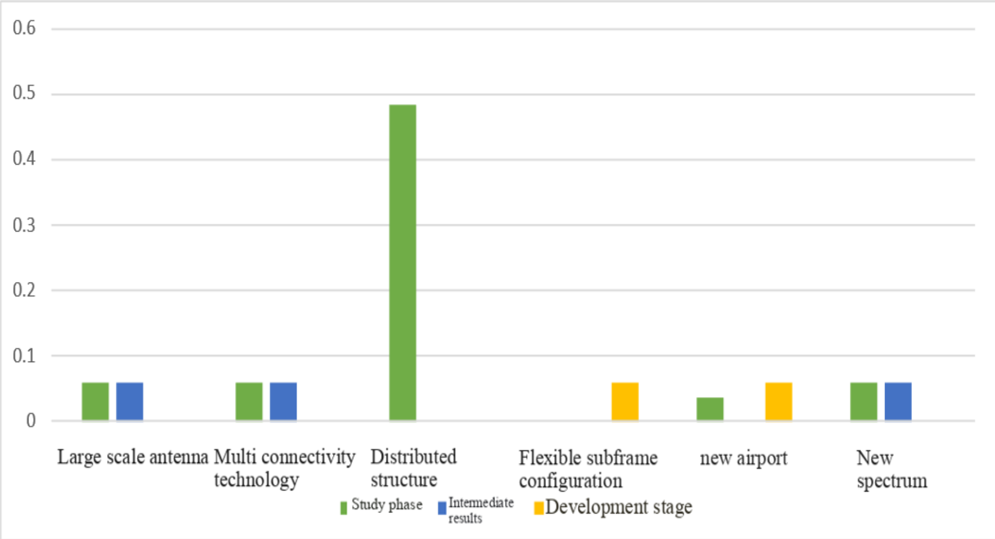

Fig. 5: When $w_{it} = 400$, Optimal Portfolio Strategy in the Field of 5G Communication Technologies

The findings demonstrate that an optimal investment strategy in communication technologies must be comprehensively structured for each key technological domain, taking into account government subsidies, required return thresholds, investment risks, and exit costs. Within this configuration, distributed cloud architecture emerges as a central focus for late-entrant firms seeking to close the performance gap with established industry leaders. For these firms, sustained engagement in independent R&D constitutes a vital strategic mechanism for strengthening competitiveness. Such self-directed innovation enables latecomers to reconcile the dual imperatives of maximising net investment returns while simultaneously containing potential risks.

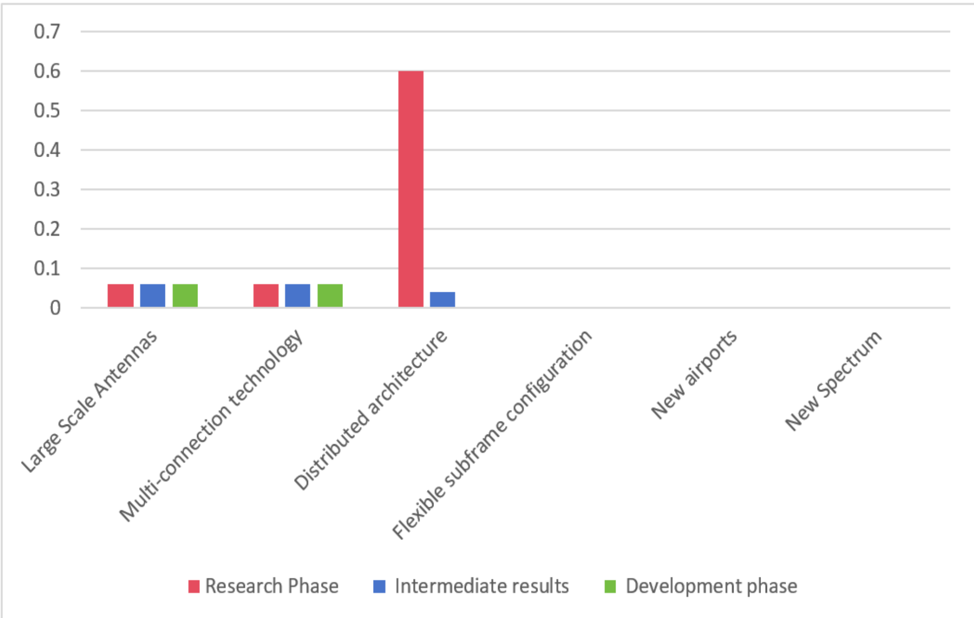

Fig. 6: Optimal Portfolio Strategy in 5G Communication Technologies When $w_{it} = 500$

4.3 Changes in the Net Return on Investment Portfolio of Latecomer Firms

This section examines variations in the net portfolio returns of late-entrant firms across different risk levels once government subsidies are incorporated, as illustrated in Figure 7. The results highlight the robustness of the optimal allocation strategy, which accommodates uncertainties inherent in investments in core communication technologies. This robustness is achieved by systematically categorising these technologies and translating R&D uncertainties into quantifiable risk indicators. Figure 7 further illustrates that elevated risk does not inherently generate higher returns, thereby challenging the simplistic assumption of a linear risk–profit relationship. From a broader perspective on R&D investment, much of the risk arises from managerial decision-making. When communication experts are tasked with forecasting expected returns and assigning risk values across technology domains and timeframes, firms can effectively transform uncertainty into measurable variables. Consequently, investment strategies must carefully balance revenue targets with acceptable risk thresholds, recognising that returns tend to plateau once risks exceed certain limits.

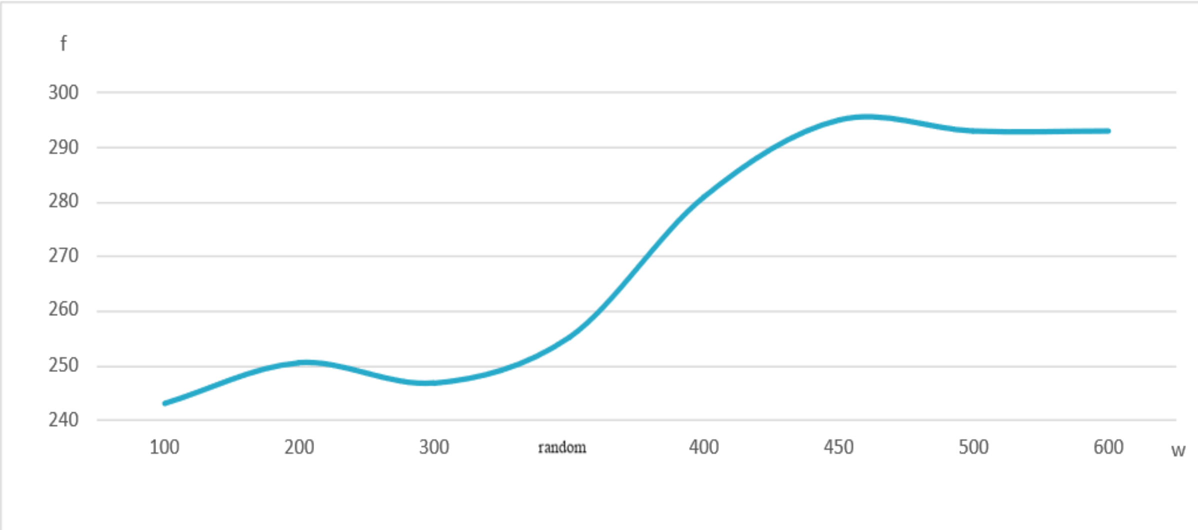

Fig.7: Changes in Net Portfolio Returns of Latecomer Firms with Different w_{it}

Accordingly, the portfolio strategy shaped by enhanced government subsidies can stimulate R&D investment while simultaneously strengthening patent quality. Subsidies alleviate financial constraints, granting firms greater capacity to channel resources into research, thereby increasing not only the volume of patents but also their technological sophistication, which enhances competitiveness in the patent race. Moreover, subsidies enable firms to diversify their investment portfolios, spreading resources across multiple technological domains and mitigating the risk associated with reliance on a single project. This diversification broadens the scope for potential breakthroughs in patent development. In addition, because subsidies are often aligned with policy priorities, firms tend to recalibrate their investment portfolios in line with governmental directions. For instance, if subsidy frameworks favour the integration of 5G with artificial intelligence and the Internet of Things, enterprises are likely to redirect resources accordingly. Finally, subsidy mechanisms also facilitate long-term technological planning, equipping firms with the financial resilience and temporal flexibility to engage in forward-looking strategies such as early-stage exploration of 6G and other emerging communication technologies. Such forward positioning can help latecomer firms compensate for disadvantages in the current 5G patent race.

4.4 Case Study-Technology Portfolio Reconstruction of China Interchina Xutron in Response to the "East-West Compute Transfer Project" Project Strategy Research

The case of Zhongji Xutron (CIIC) is presented as a representative example that vividly encapsulates the theoretical mechanisms advanced in this study. It demonstrates how a leading enterprise strategically reconfigures its technology investment portfolio in direct response to national policy initiatives, particularly the state-led programme known as the 'East-West Computing Transfer' project, together with associated government subsidies. Although geographically situated within the Chinese context, the strategic dilemma it illustrates—how firms leverage policy-induced resources to optimise R&D investments across competing technological pathways under uncertainty—is a universal challenge confronting high-technology enterprises globally. The CIIC case thus provides a substantive context for grounding the mathematical framework developed in this paper, highlighting its practical relevance and explanatory robustness.

It is important to emphasise that the CIIC case study is not intended to statistically validate the model but rather to serve as an illustrative narrative that conveys how the core mechanisms unfold in practice. In 2022, the Chinese government launched the 'East Counts West Counts' initiative, a nationwide infrastructure programme aimed at balancing the national distribution of computing power by directing the growing demand for high-capacity computing in the eastern regions to the western part of the country. The policy emphasises that western computing nodes must be dominated by domestically produced and independently controllable technologies, with state-owned capital leading the construction of western computing hubs. In addition, the government supports participating enterprises through preferential subsidies and favourable resource deployment, including land allocation and energy provision.

Within this context, optical modules represent the core components of optical communication systems. They perform the critical function of converting electrical signals into optical signals and vice versa, thereby enabling high-capacity data transmission across networks. The rise of artificial intelligence and large-scale models has further intensified the need for robust computing power, making optical communication networks the foundational infrastructure upon which such computational demands rest. Consequently, optical modules occupy a central position in enabling the long-term development of advanced communication technologies. As one of the principal suppliers in the optical module industry, CIIC responded proactively to policy directives by restructuring its technological portfolio to align with government priorities. The policy explicitly mandated a higher localisation rate for optical modules in western data centres, and in response, CIIC markedly expanded its R&D expenditures. For example, while R&D investment in 2021 was limited to 541 million yuan, this increased substantially to 767 million yuan in 2022 and 739 million yuan in 2023. Notably, by the first three quarters of 2024 alone, the firm's R&D expenditure had already exceeded 742 million yuan, as illustrated in Figure 8. This upward trajectory reflects the firm's active repositioning of its innovation strategy in line with subsidy-driven incentives.

Supported by favourable policy orientation, CIIC strategically concentrated on advancing silicon optical integration technologies and accelerating the iterative upgrading of optical modules towards ultra-high-speed products, such as 800G, 1.6T, and beyond. Concurrently, the enterprise capitalised on preferential tax credits to offset R&D risks, thereby reinforcing its financial resilience in pursuing multiple parallel technological routes. These include high-speed optical modules, silicon optical integration, and co-packaged optics (CPO). Furthermore, CIIC extended its technological offerings across a wide spectrum of applications, ranging from 5G pre-transmission, mid-transmission, and backhaul modules, to transmission modules for metro, backbone, and core networks, as well as fixed network fibre access solutions (FTTX). Through the dual effect of government subsidies and preferential procurement policies, CIIC significantly enhanced its risk-bearing capacity while diversifying its innovative pathways. This case demonstrates how national policy instruments not only incentivise firms to intensify their R&D commitments but also reshape their long-term strategic trajectories, enabling them to strengthen competitiveness and actively contribute to next-generation communication technologies.

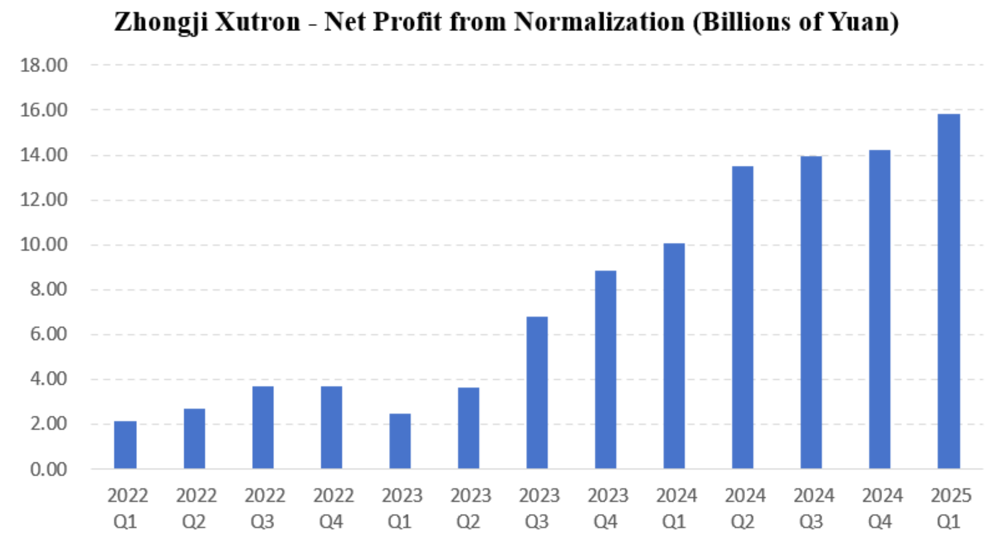

Fig.8: Trend of Net Profit Attributable to Parent Company of Zhongji Xutron (2022-2025)

Simultaneously, the government's localisation requirements compelled enterprises to accelerate breakthroughs in independently controlled core technologies. In response, CIIC expanded its patent portfolio in silicon optical integration, positioning this technology as a cornerstone of its strategic development. Silicon-based optical modules deliver distinct advantages, including lower production costs and higher gross margins relative to traditional products, thereby generating a self-reinforcing "policy–technology–market" cycle. Policy supports incentivised innovation, technological advances improved product competitiveness, and market feedback reinforced demand, thus creating a positive loop of sustained growth. This dynamic is reflected in CIIC's market performance. Bolstered by surging demand for 800G optical modules and strong growth in overseas markets, the company recorded annual revenues of 23.862 billion yuan in 2024, representing a 122.64 per cent year-on-year increase. Concurrently, net profits rose to 5.171 billion yuan, reflecting a year-on-year growth rate of 137.93 per cent, as illustrated in Figure 9. Such financial outcomes demonstrate how strategic alignment with state-led policy incentives translates into both technological leadership and superior market returns.

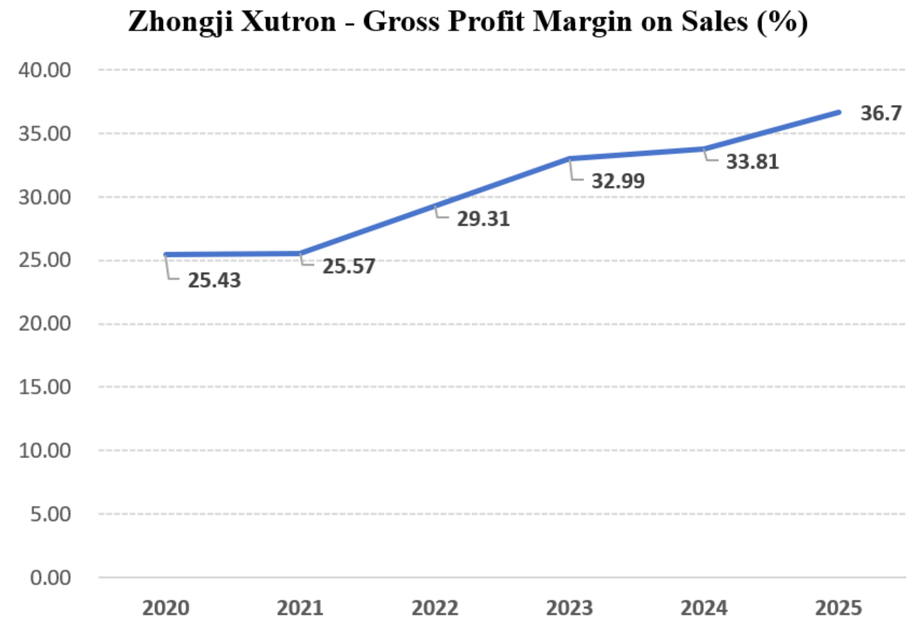

Fig.9: Trend of Sales Gross Margin Change of Zhongji Xutron (2020-2025)

Although the primary case is situated within China, analogous strategic patterns can be observed in firms responding to subsidy-driven frameworks such as the United States CHIPS Act and the European Union's Green Deal. This suggests that the insights derived from the model possess cross-institutional applicability. In CIIC's case, as a mature-stage enterprise, the subsidy mechanism effectively reduced marginal costs—tax credits covered up to 30 per cent of western investment expenditures—thereby directly enhancing the firm's risk tolerance. This, in turn, facilitated diversification across multiple technological domains, validating the practical significance of examining how government subsidies shape R&D portfolio configurations. Overall, the CIIC experience illustrates how policy-induced incentives function as leverage points, prompting enterprises to reconfigure their R&D investment portfolios in ways that not only mitigate risk but also accelerate technological innovation and improve performance outcomes.

5. Summary

5.1 Policy and Institutional Implications

The implications of this study transcend firm-level strategy and extend to the broader domains of policymaking and institutional architecture. The findings demonstrate that the effectiveness of government subsidies is determined not solely by their absolute magnitude but equally by their structural design. Specifically, differentiated or tiered subsidy schemes—targeting distinct stages of the innovation cycle—can be more effective than uniform allocations. By directing higher support toward early-stage, high-risk research domains, such as distributed cloud architecture in the simulation, policymakers can stimulate foundational breakthroughs that form the basis for subsequent technological advances. Equally significant is the de-risking function of subsidies, which constitutes a form of public good. By reducing the inherent uncertainties of R&D investment, subsidies lower entry barriers, enabling a wider range of firms to engage in patent competition. This broadens participation, fosters diversity within the national innovation portfolio, and enhances systemic resilience against external shocks. As such, subsidies should not be evaluated exclusively in terms of their immediate return on investment for individual enterprises but rather in terms of their contribution to strengthening the long-term capacity of the innovation ecosystem. From this perspective, subsidy frameworks should be designed as strategic policy instruments that not only incentivise firm-level investment but also cultivate a self-sustaining, diversified, and competitive national innovation system. This systemic orientation enhances a country's ability to withstand global technological competition and navigate market volatility, thereby reinforcing innovation policy as an integral dimension of economic policy.

5.2 Conclusion

Exploring risky investment portfolios with government subsidies in simulated patent competition is vital for communication enterprises seeking to optimise resource allocation and strengthen competitiveness. Risk diversification remains central to high-risk technology research and development, where patent victories offer advantages, but outcomes extend beyond patents alone. Firms must therefore evaluate expected returns comprehensively to mitigate losses and maximise gains. This study develops a bi-objective planning model integrating subsidies, returns, risks, and replacement costs to guide latecomer firms in identifying optimal portfolio strategies. By classifying and assessing key technologies, the model converts R&D uncertainty into measurable indicators, enabling informed investment decisions. For disadvantaged firms, effective strategies involve not only catching up with industry leaders but also diversifying patent layouts, selecting investments prudently, and pursuing risk-adjusted returns. Beyond 5G, the framework is applicable to other sectors such as artificial intelligence, renewable energy, and biopharmaceuticals, where similar tradeoffs exist between competing innovations. Such cross-sectoral applications validate the model's generalisability and yield comparative insights into innovation dynamics. Findings suggest that well-structured subsidies can enable lagging firms to engage in foundational, high-risk research, broadening the technological base, fostering spillovers, and avoiding premature lock-in. Strategic positioning further guides patent portfolio choices, ensuring alignment with both long-term goals and short-term challenges. This research provides theoretical and practical support for communication enterprises navigating patent competition, helping them achieve sustainable growth and global technological leadership. For China, enhancing patent strategy and management is essential to realise value, gain advantages, and compete effectively in the global arena.

Author Contributions

Conceptualization, J.W.; methodology, J.W.; software, C.X.; validation, J.W. and C.X.; formal analysis, C.X.; investigation, J.W.; resources, C.X.; data curation, J.W.; writing—original draft preparation, J.W.; writing—review and editing, J.W.; visualization, C.X.; supervision, C.X.; project administration, J.W. All authors have read and agreed to the published version of the manuscript.

Data Availability Statement

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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